Covariance, Dynamics and Symmetries, and Hadron Form Factors

Craig D. Roberts

`cdroberts@anl.gov`

[Physics Division](#)

[Argonne National Laboratory](#)

http://www.phy.anl.gov/theory/staff/cdr.html

QCD’s Challenges

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- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

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QCD's Challenges

Understand Emergent Phenomena

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- QCD – Complex behaviour

  arises from apparently simple rules

Dichotomy of Pion
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Current Algebra . . . 1968

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Highly Nontrivial

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

Intranucleon Interaction

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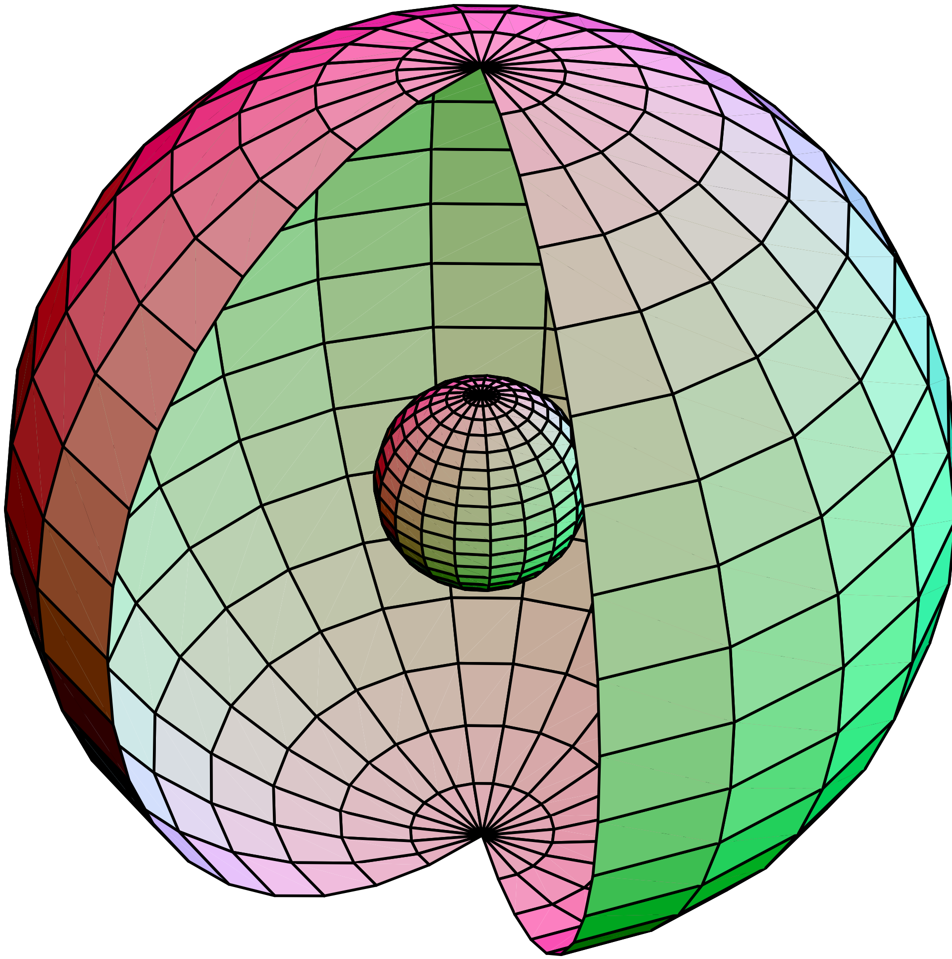

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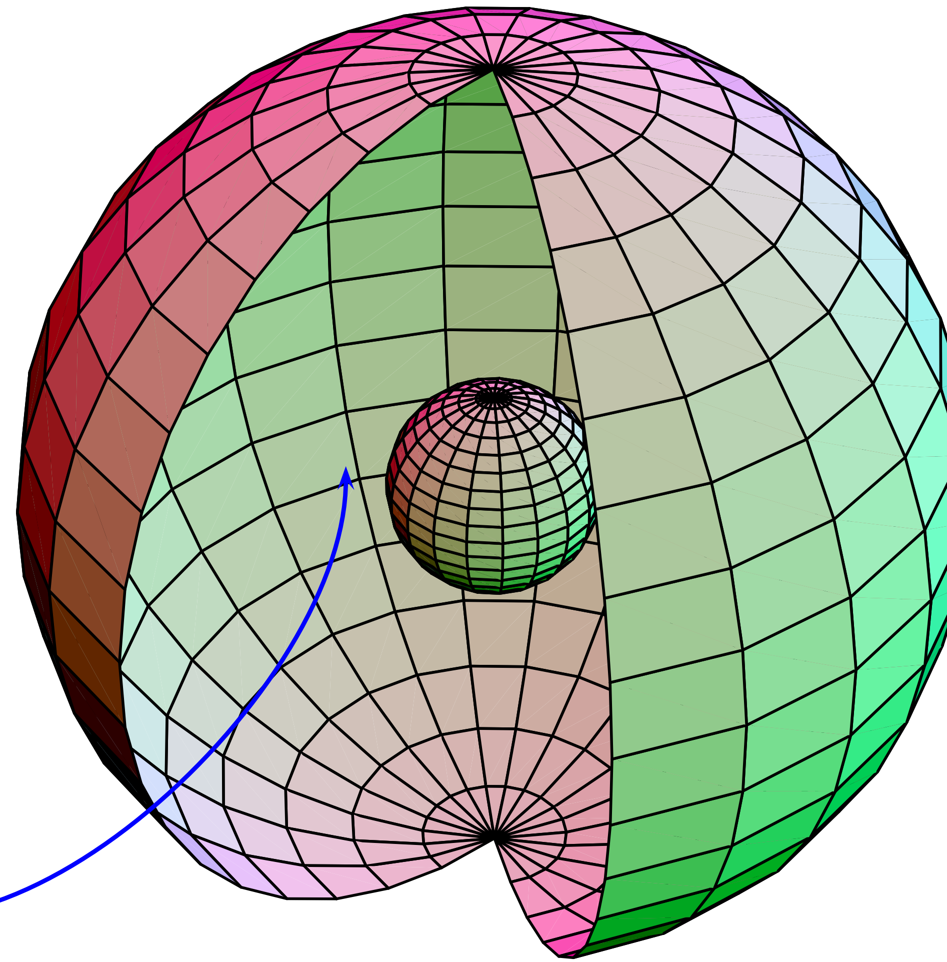
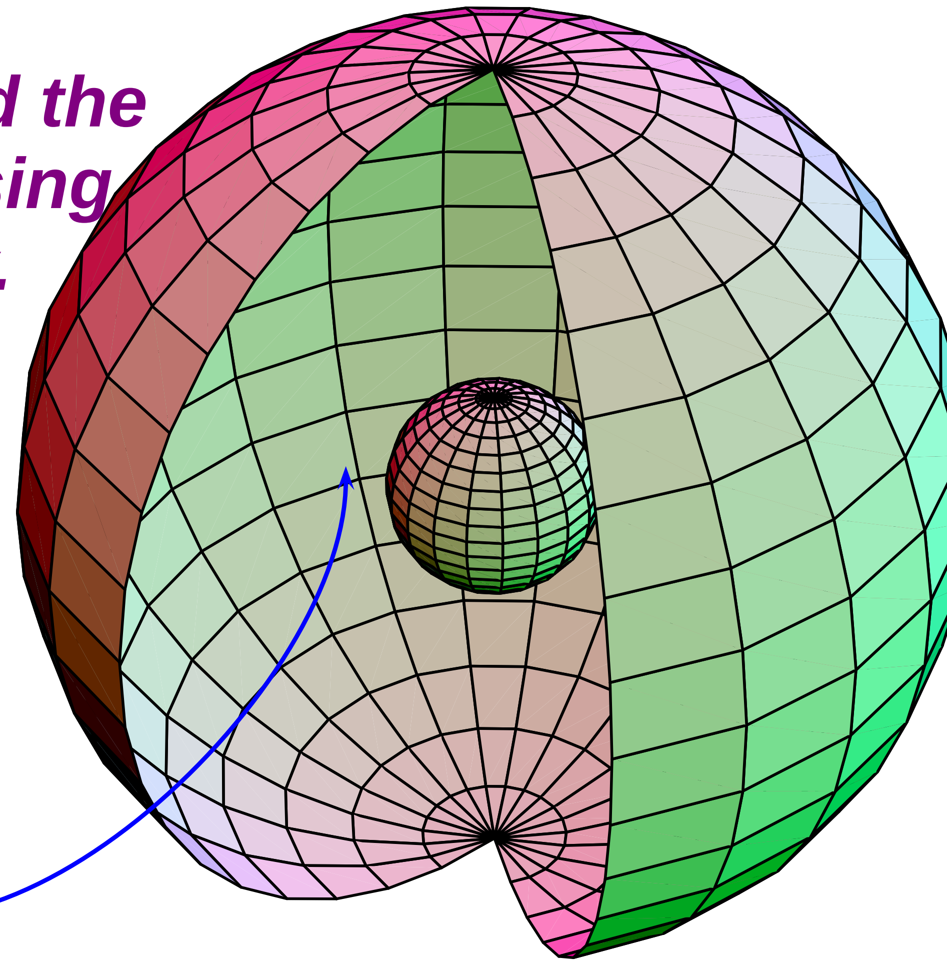


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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- $\Rightarrow$ Understanding InfraRed (long-range)
  . . . . behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

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  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
- Proving fruitful.

World . . .

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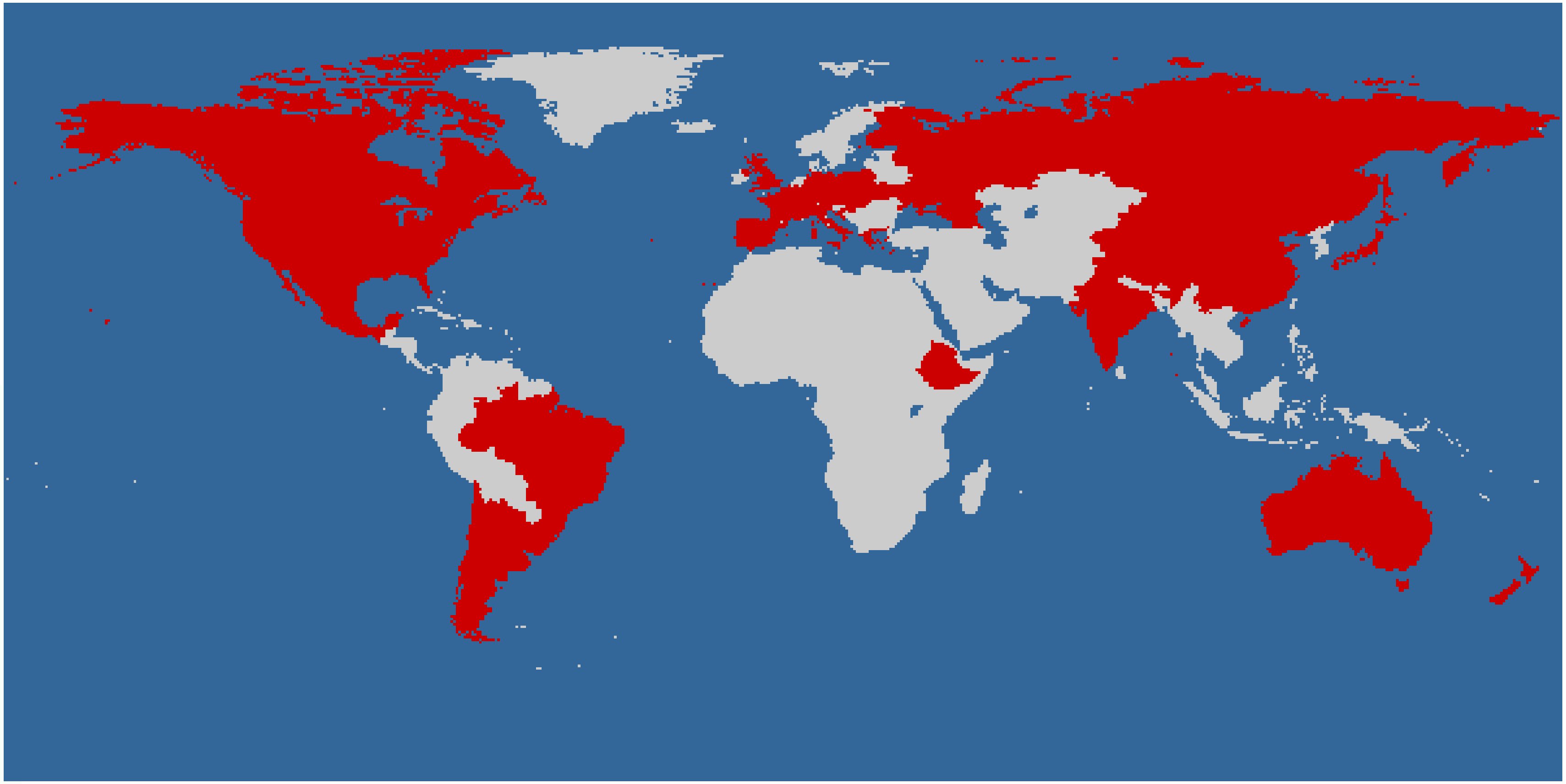
DSE Perspective



Persistent Challenge

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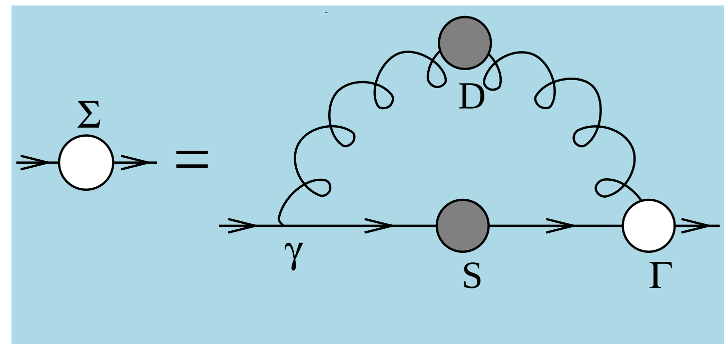

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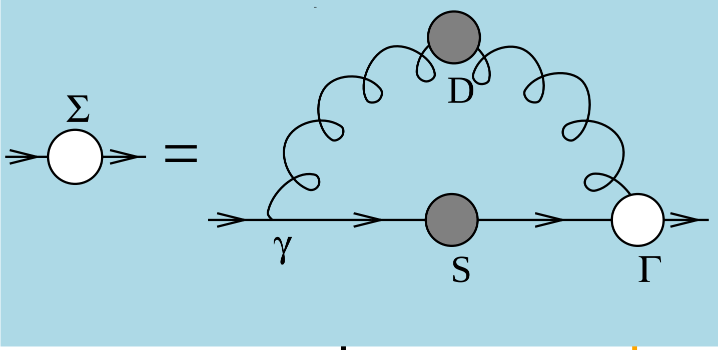

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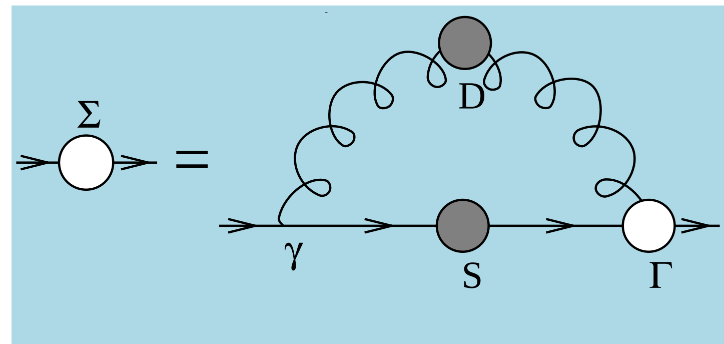

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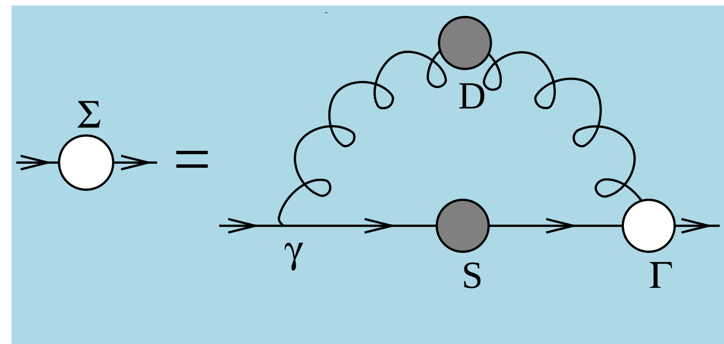

- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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Perturbative Dressed-quark Propagator

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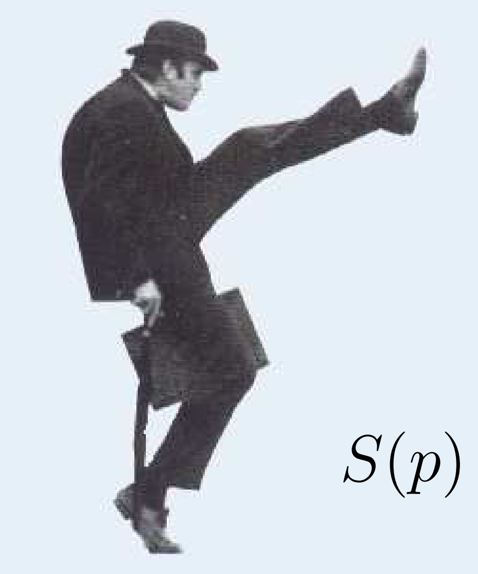

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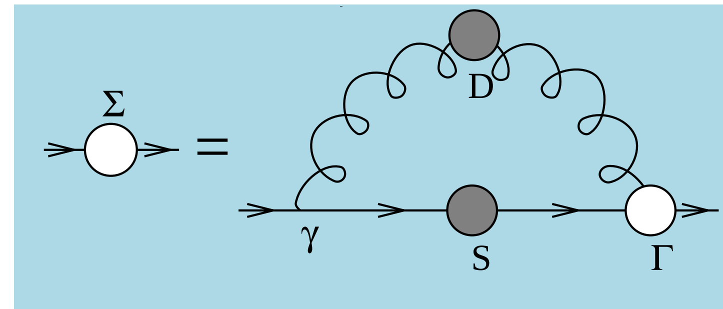

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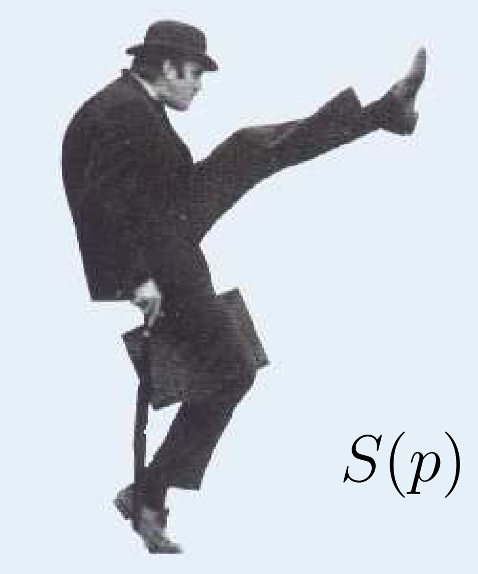

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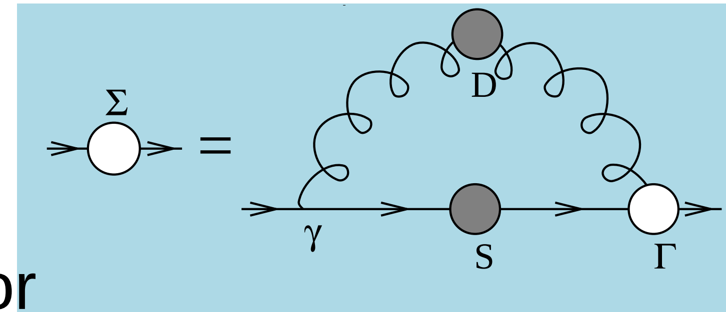

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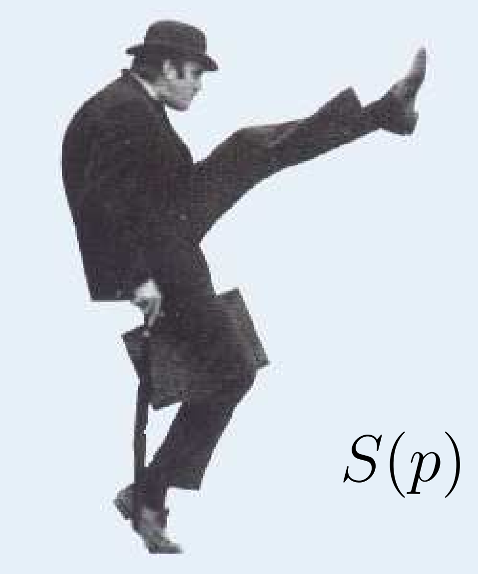

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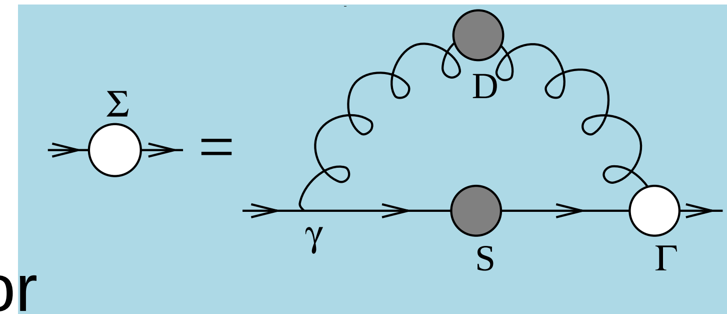

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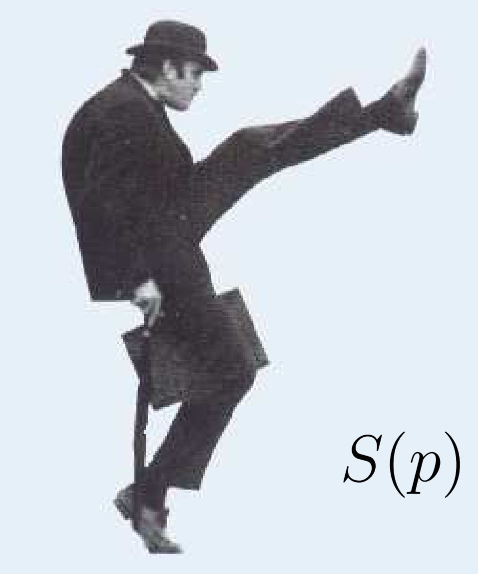

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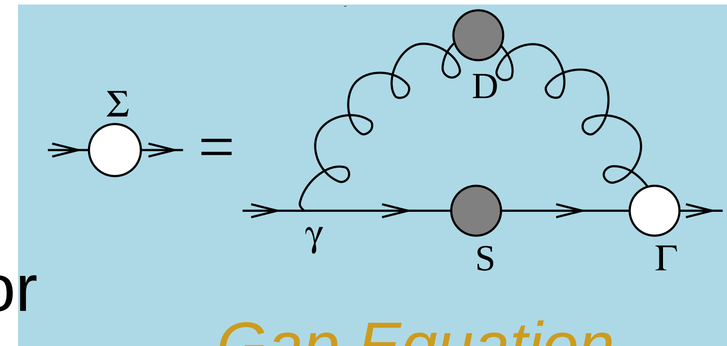

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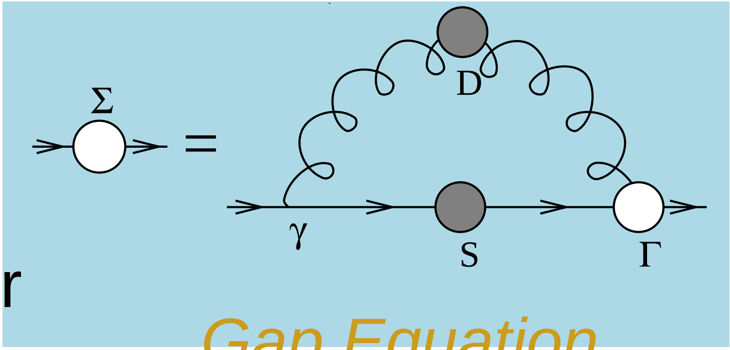

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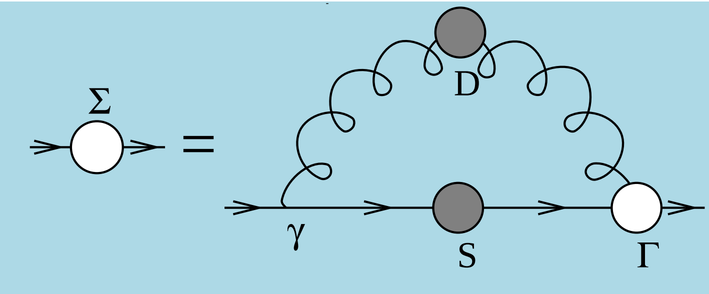

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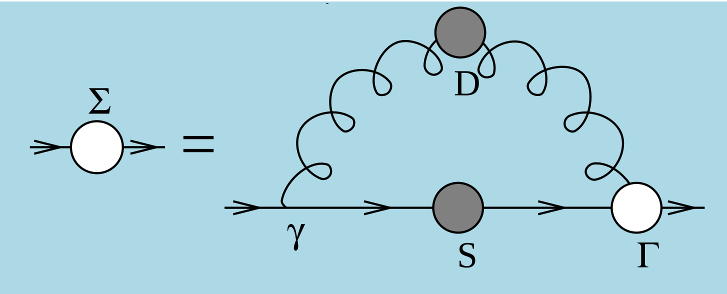

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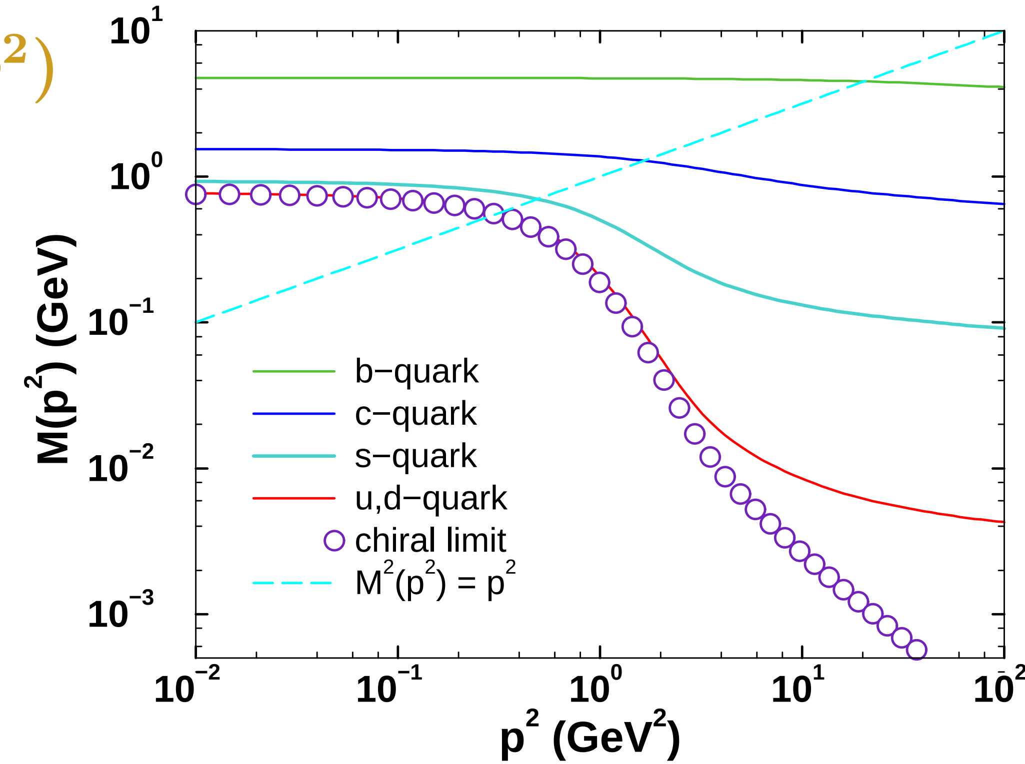

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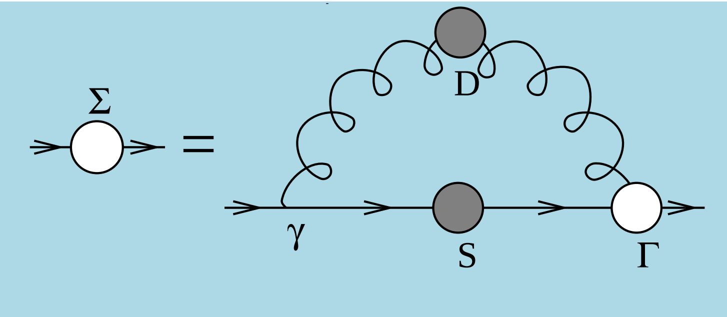

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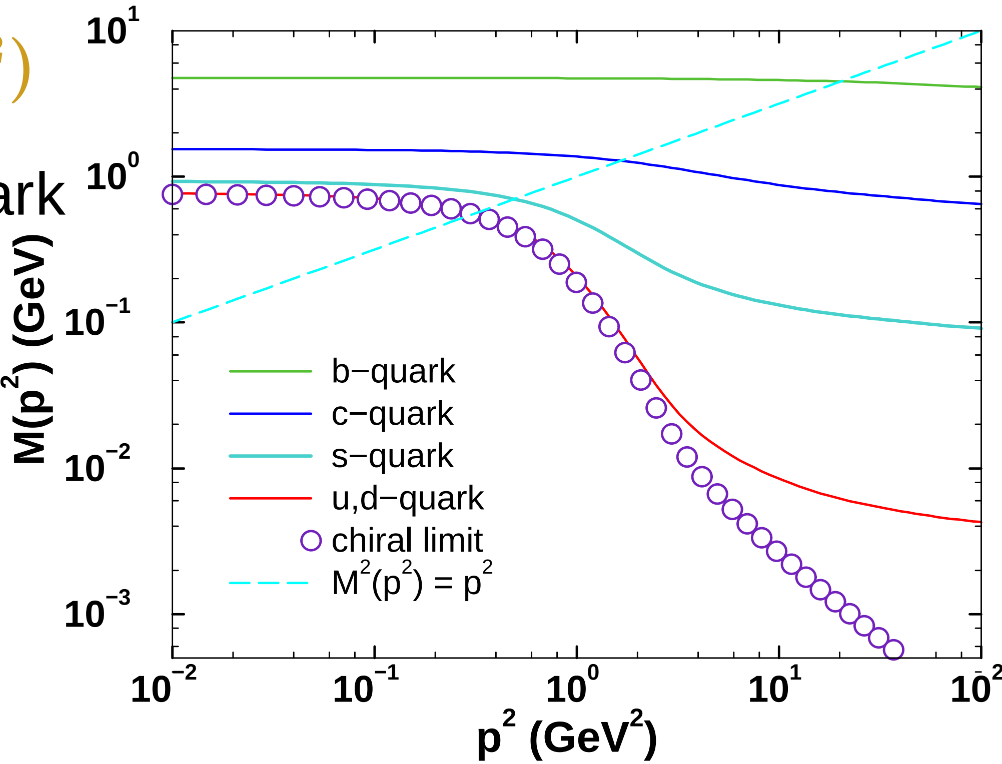
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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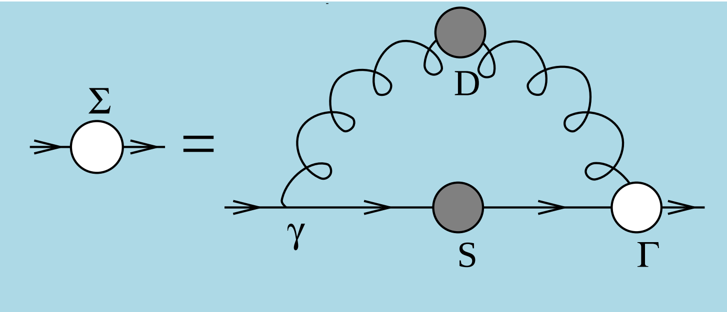

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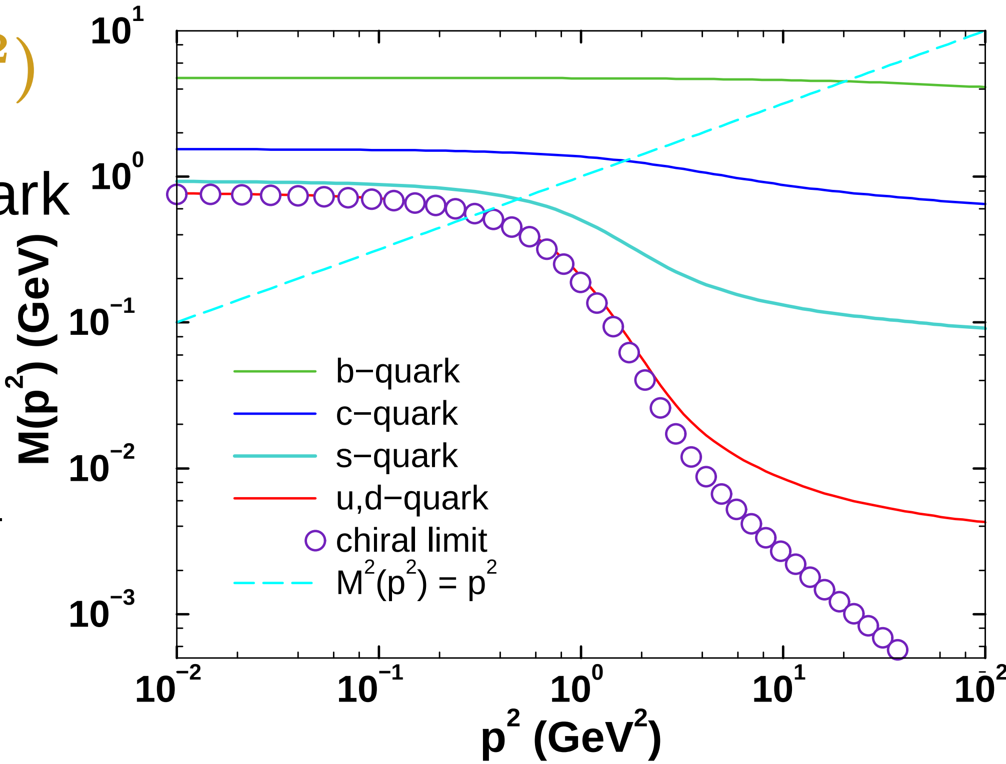

Predictions confirmed in numerical simulations of lattice-QCD

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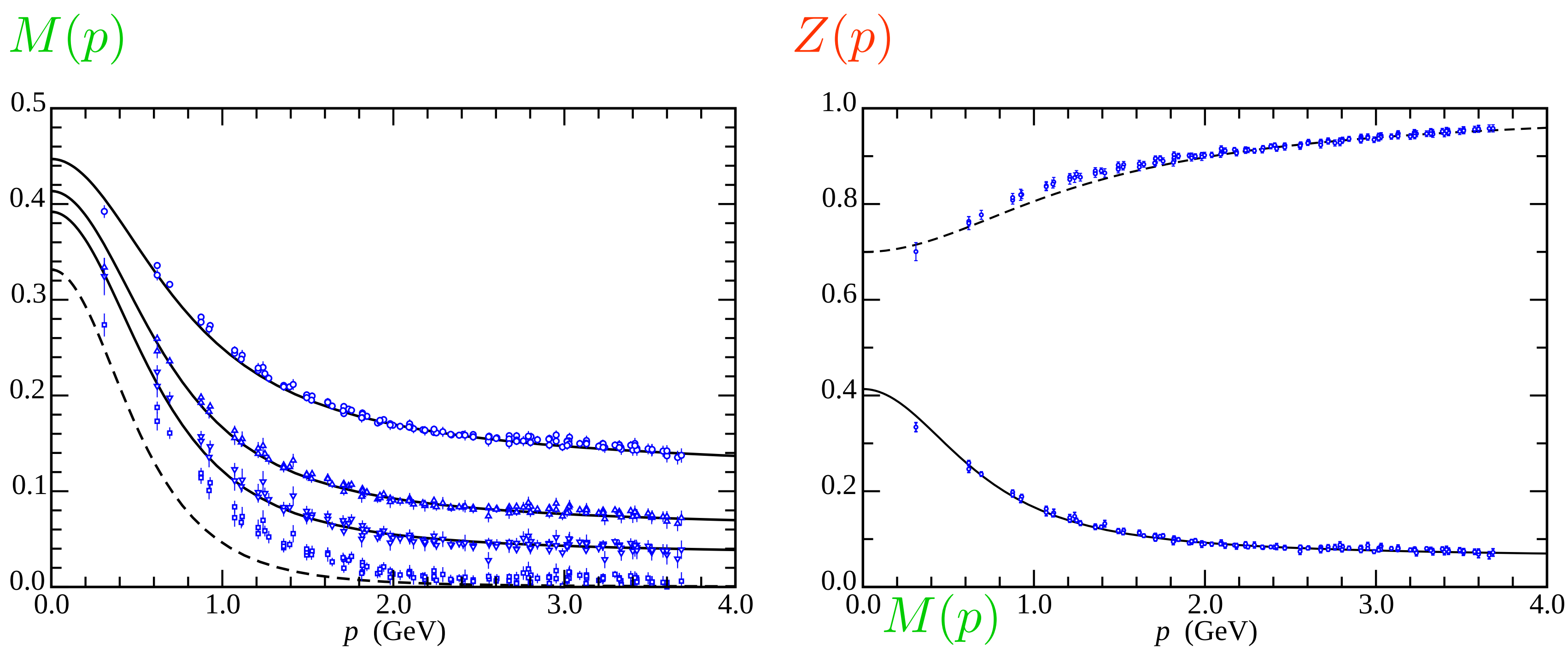
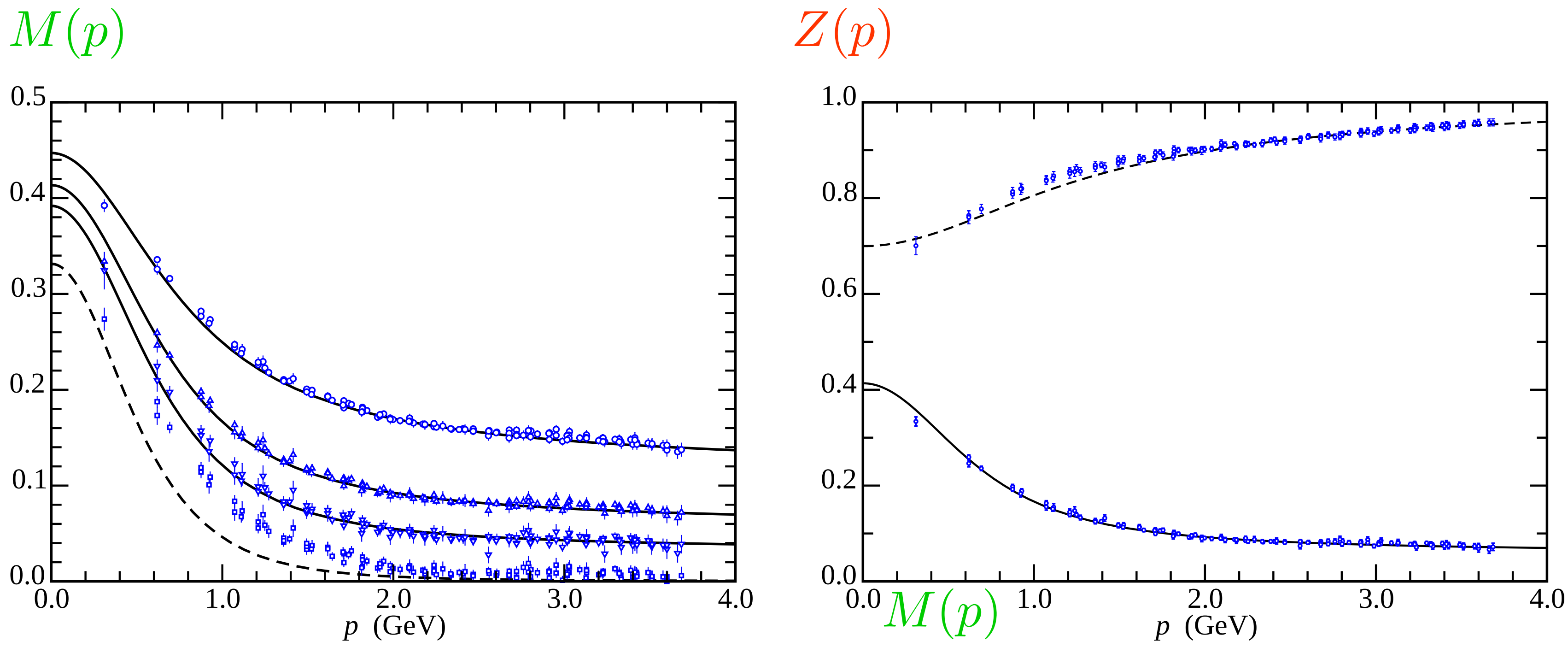
Quenched-QCD Dressed-Quark Propagator



2002 Quenched-QCD Dressed-Quark Propagator



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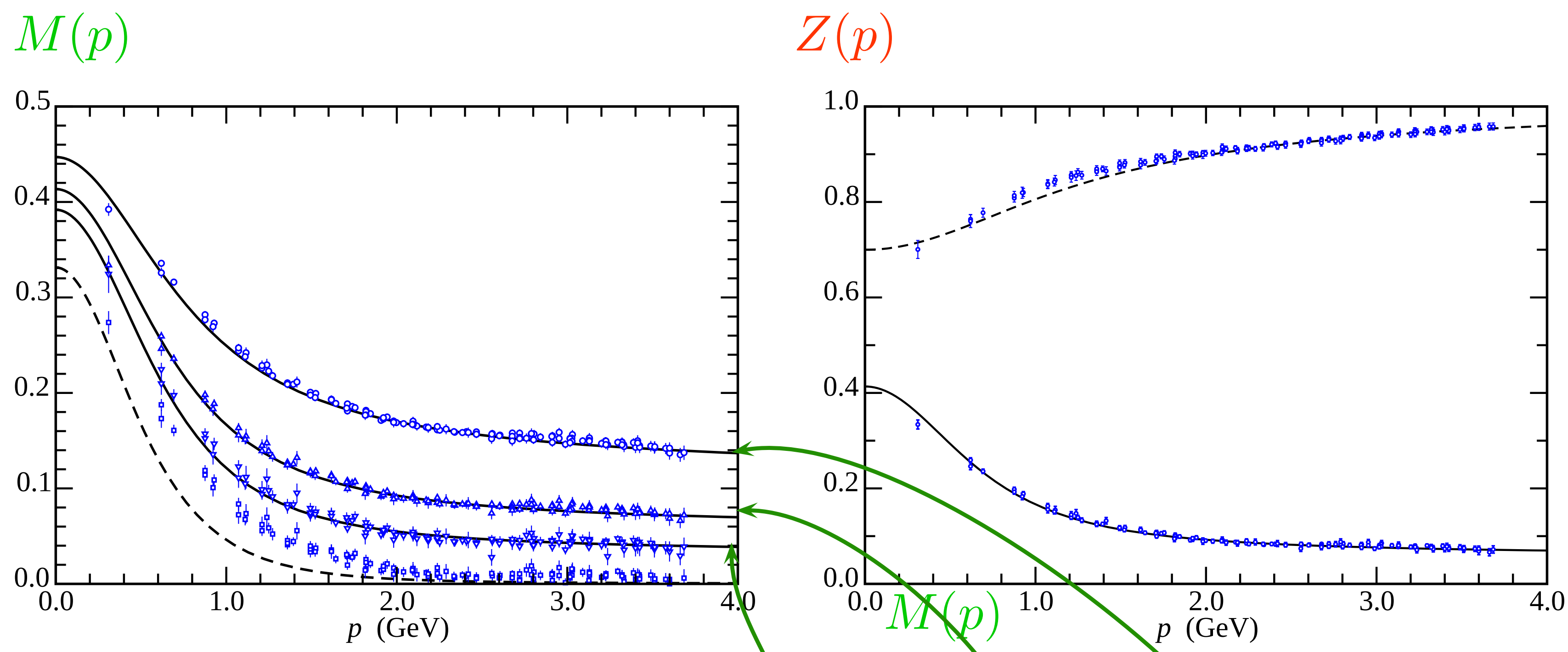

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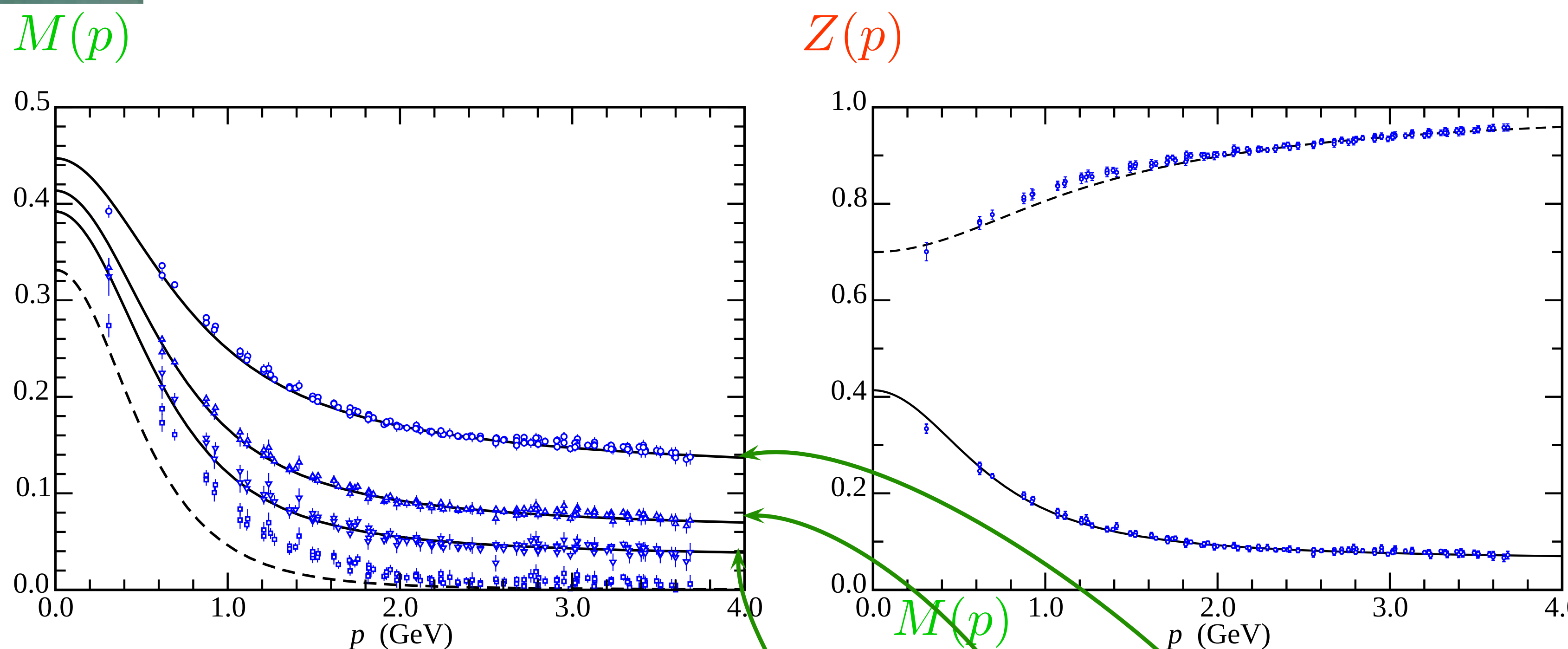

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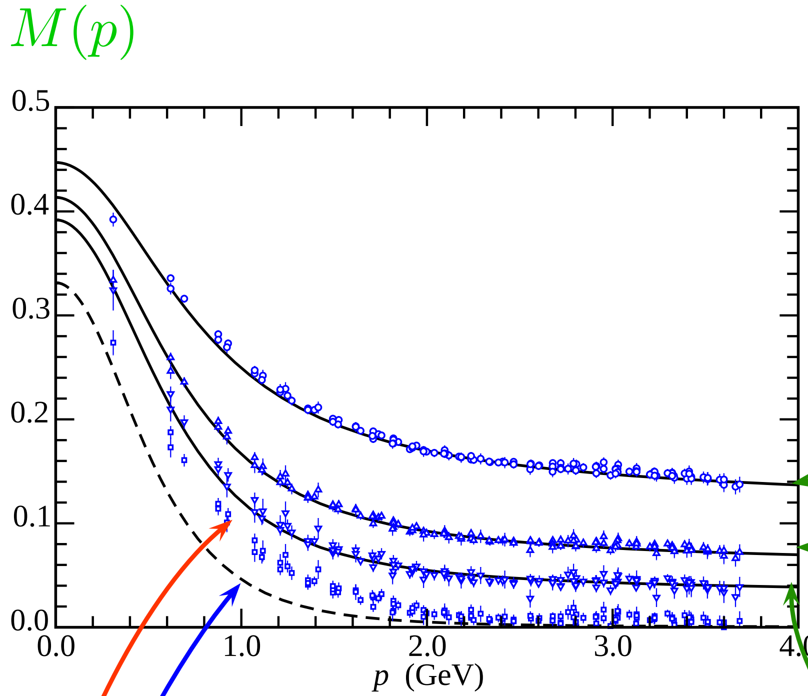

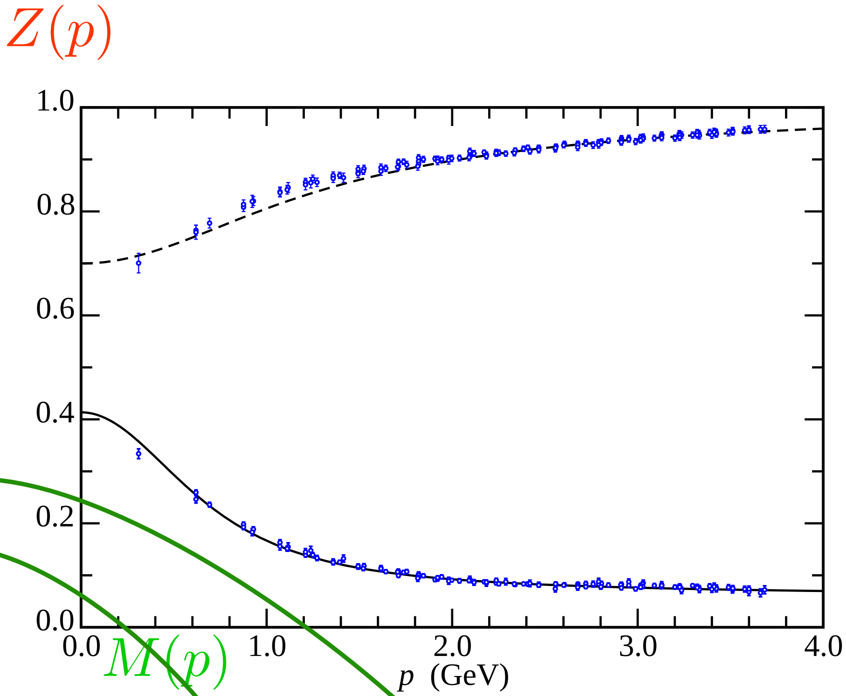

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- Linear extrapolation of lattice data to chiral limit is inaccurate

Frontiers of Nuclear Science:
A Long Range Plan (2007)

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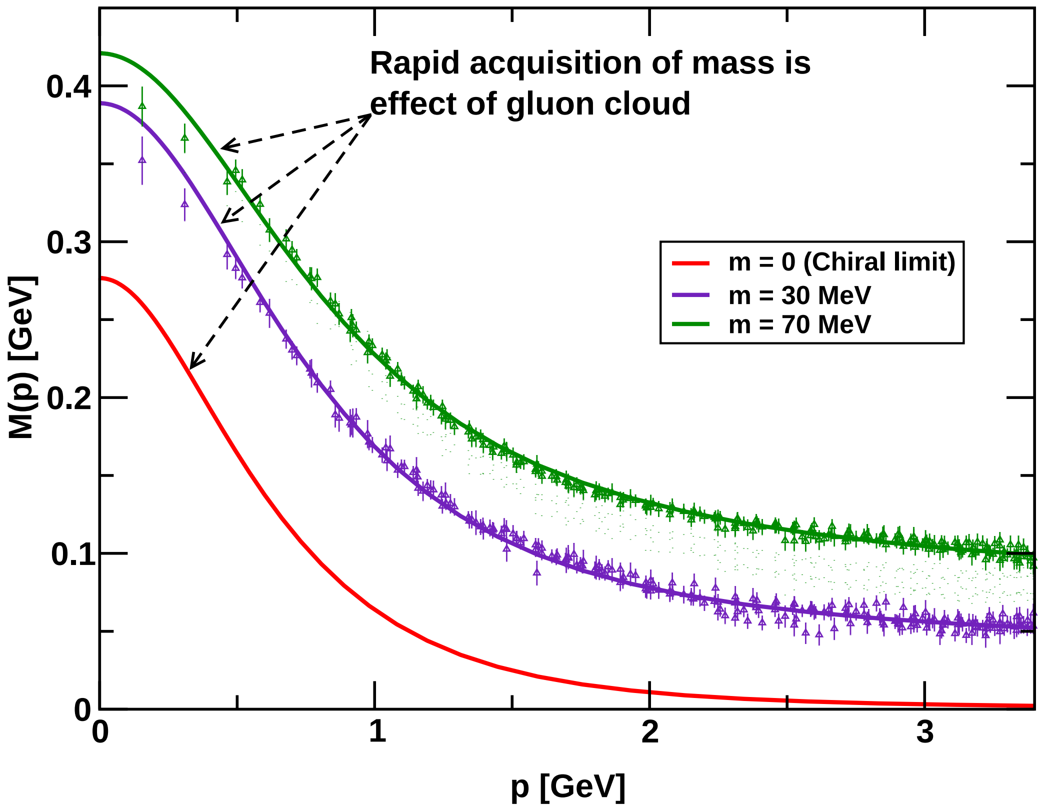
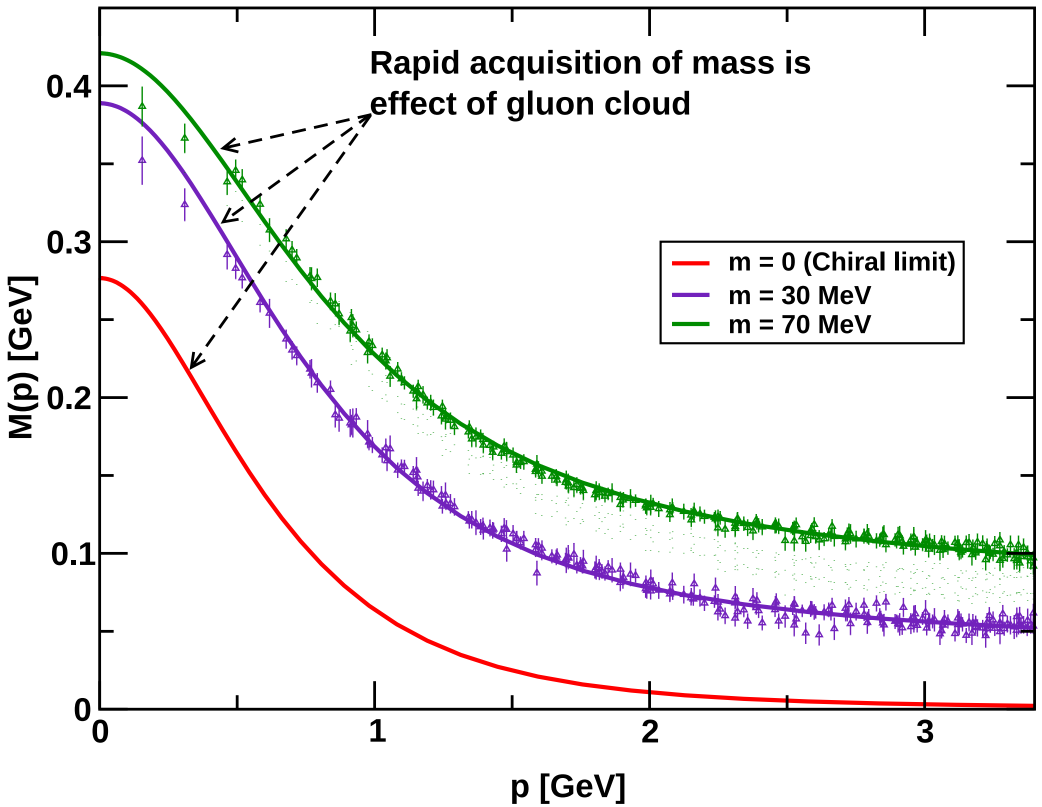


Rapid acquisition of mass is effect of gluon cloud
m = 0 (Chiral limit)
m = 30 MeV
m = 70 MeV
M(p) [GeV]
p [GeV]
0
0.1
0.2
0.3
0.4
1
2
3

Mass from nothing. In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

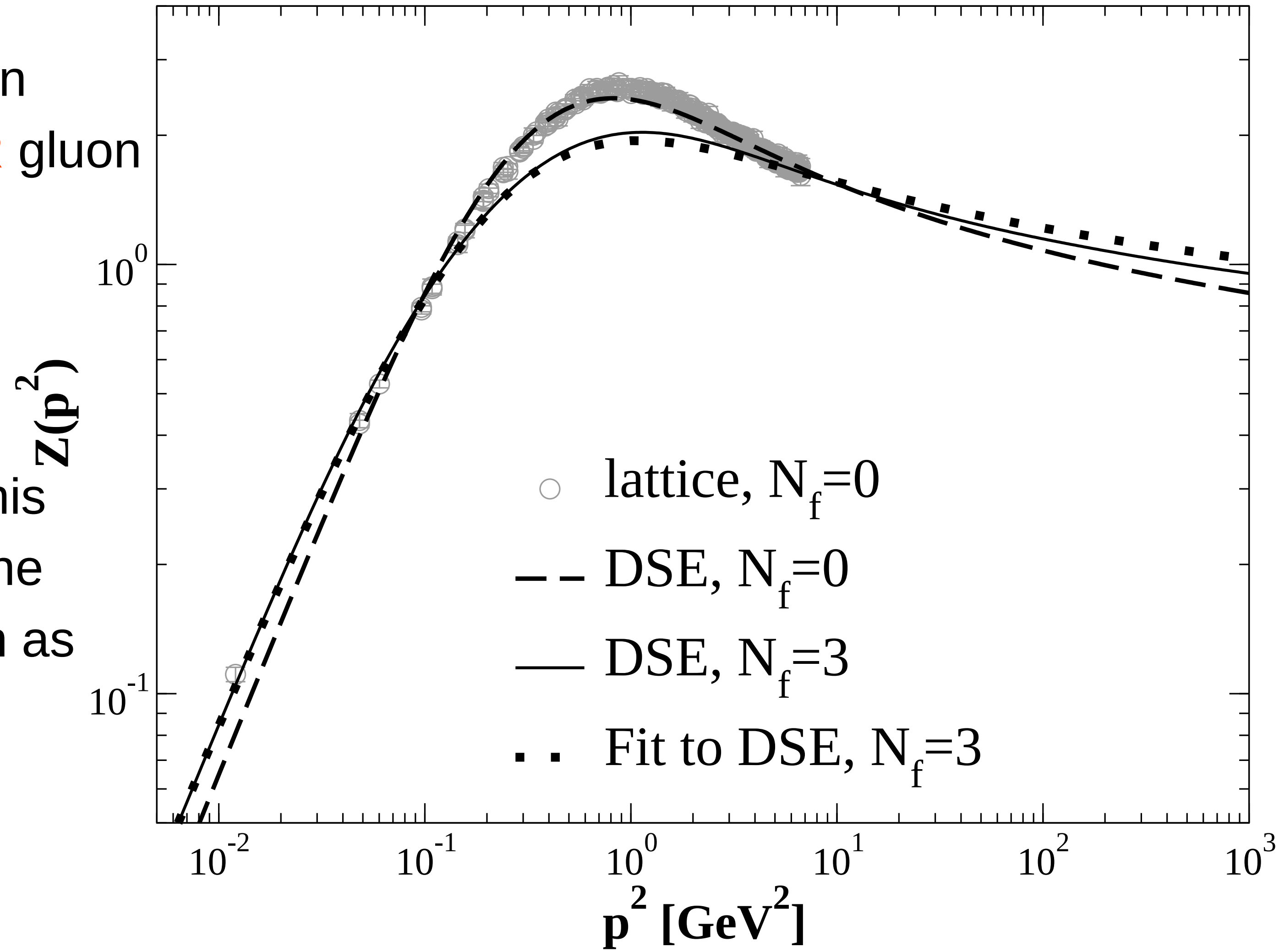
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\text{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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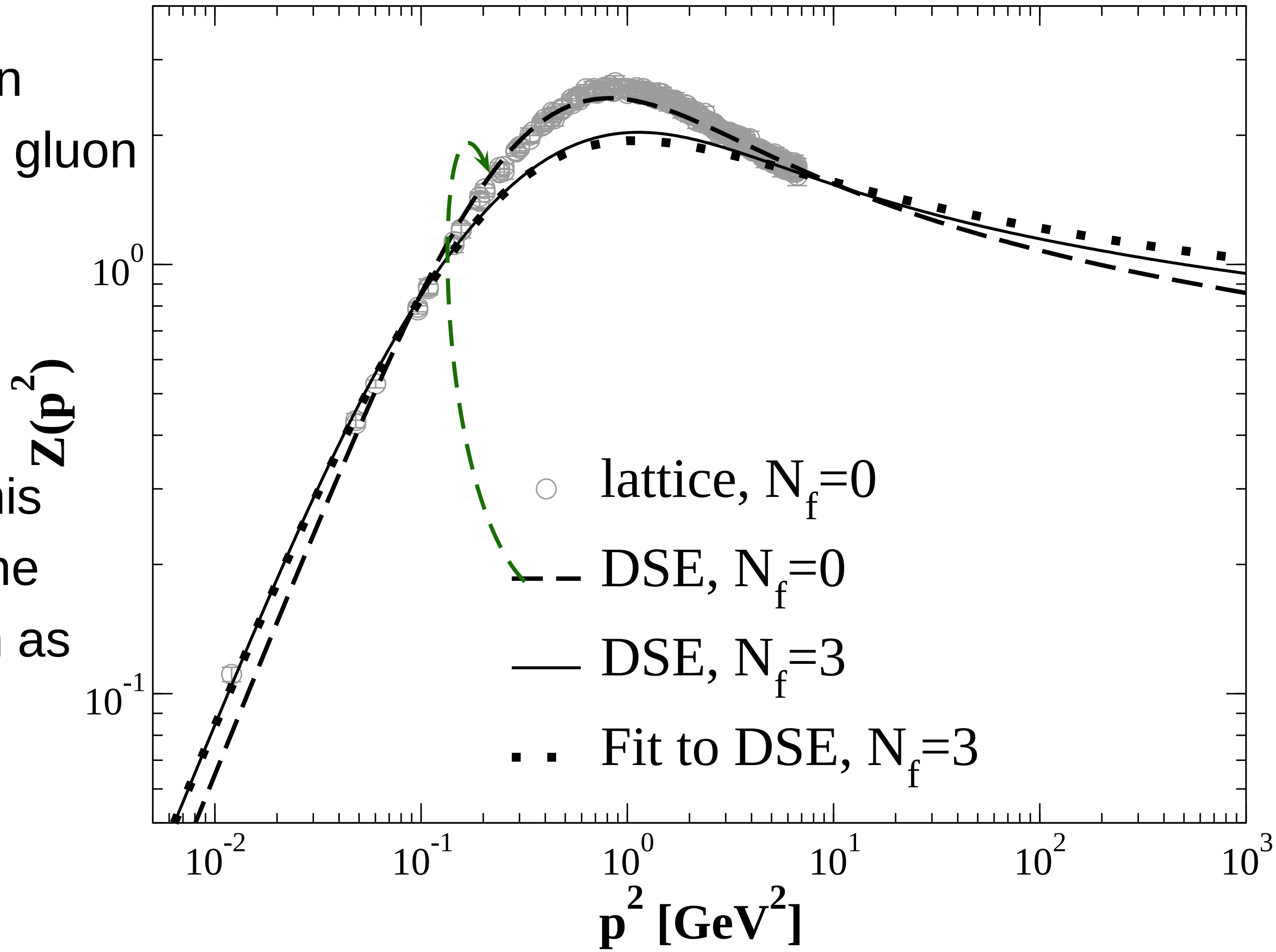

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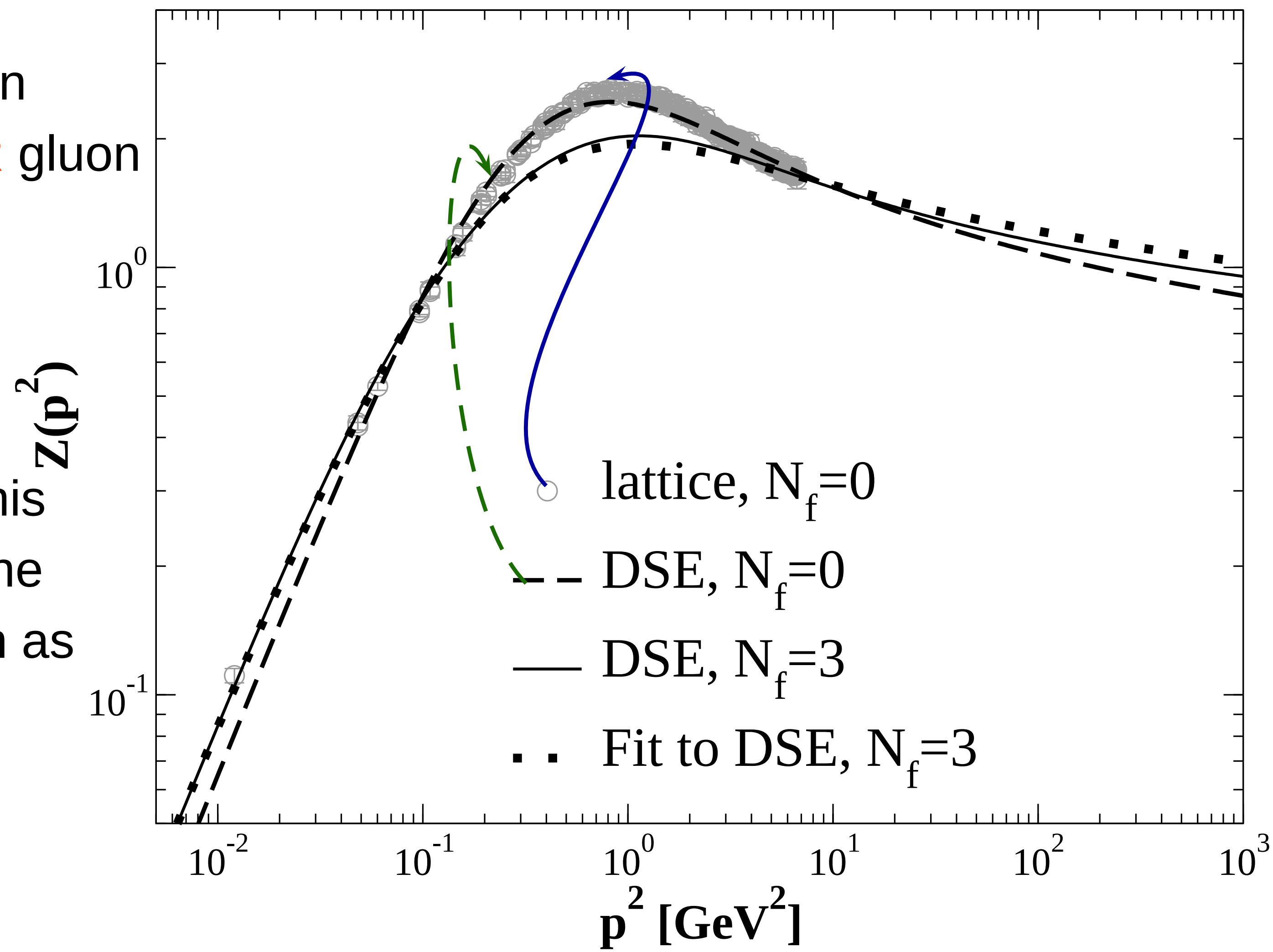

Critical Mass for Chiral Expansion

Lei Chang, *et al*., nucl-th/0605058

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- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:
$$\boxed{M(p^2; m = 0) \neq 0}.$$

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$$\boxed{M(p^2; m=0) \neq 0}.$$

- Does this mass function have a ***convergent*** expansion in current-quark mass about its nonzero chiral-limit value:

$$M(0;m) = M(0,0) + m \left. \frac{\partial}{\partial m} M(0;m) \right|_{m=0} + \ldots ?$$

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- $$M(0;m) = M(0,0) + \sum_{n=1}^{\infty} m^n a_n$$

Radius of convergence: $m_{\rm rc} = \lim_{n\to\infty} \left(\frac{1}{|a_n|} \right)^{1/n}$

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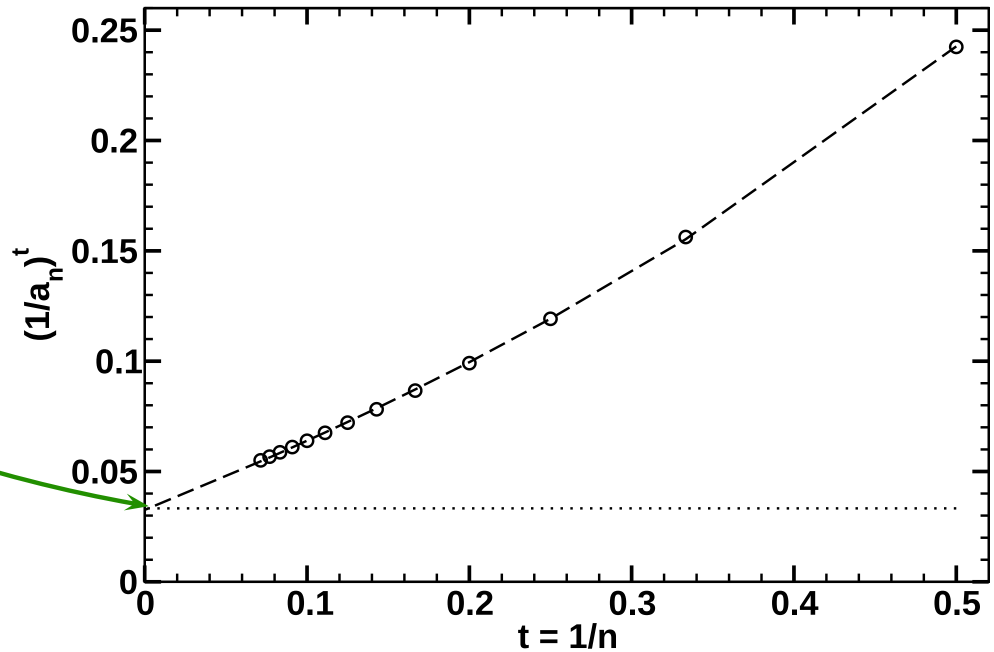
$$m_{\mathbf{rc}} = \mathbf{0.034 \pm 0.001}$$



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- Entails, e.g., lattice-QCD simulations *must* have results at $m_\pi^2 < [m_{0^-}^{\rm cr}]^2 \sim 0.2\,$GeV^2 for reasonable extrapolation via EFT.

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking ... exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta) \, \frac{\partial M_f^E}{\partial m_f(\zeta)} \, , \; (M^E)^2 := s \, | \, s = M(s)^2 \, .$$

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- Renormalisation-group-invariant and determined from solutions of the gap equation

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- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

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- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

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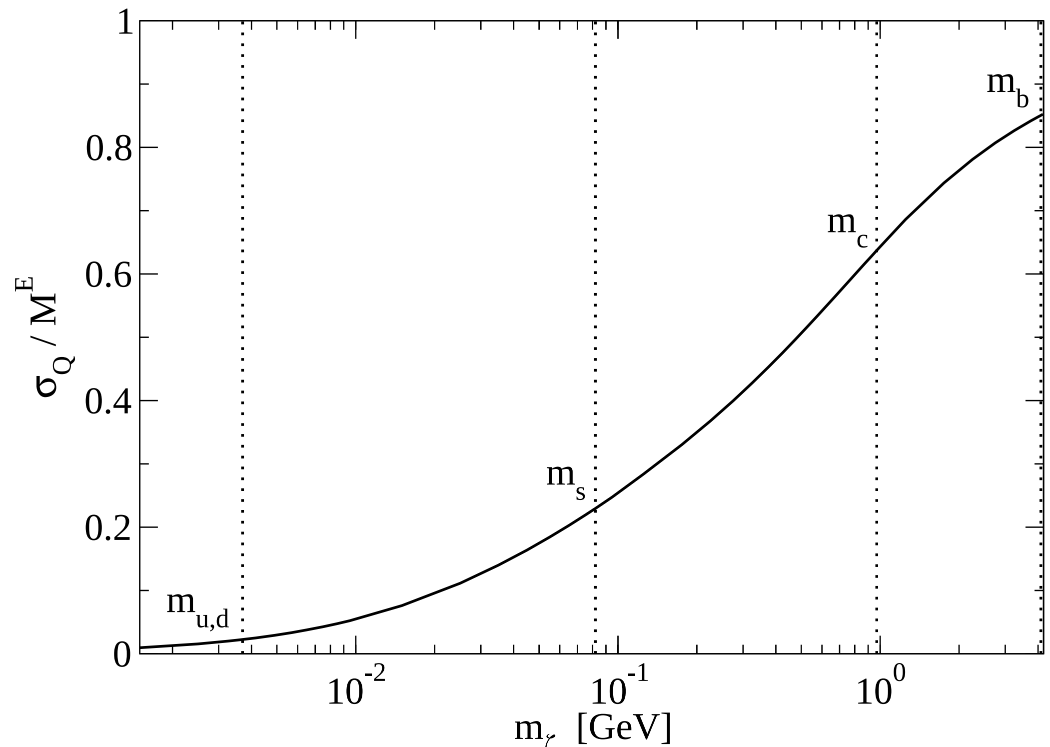

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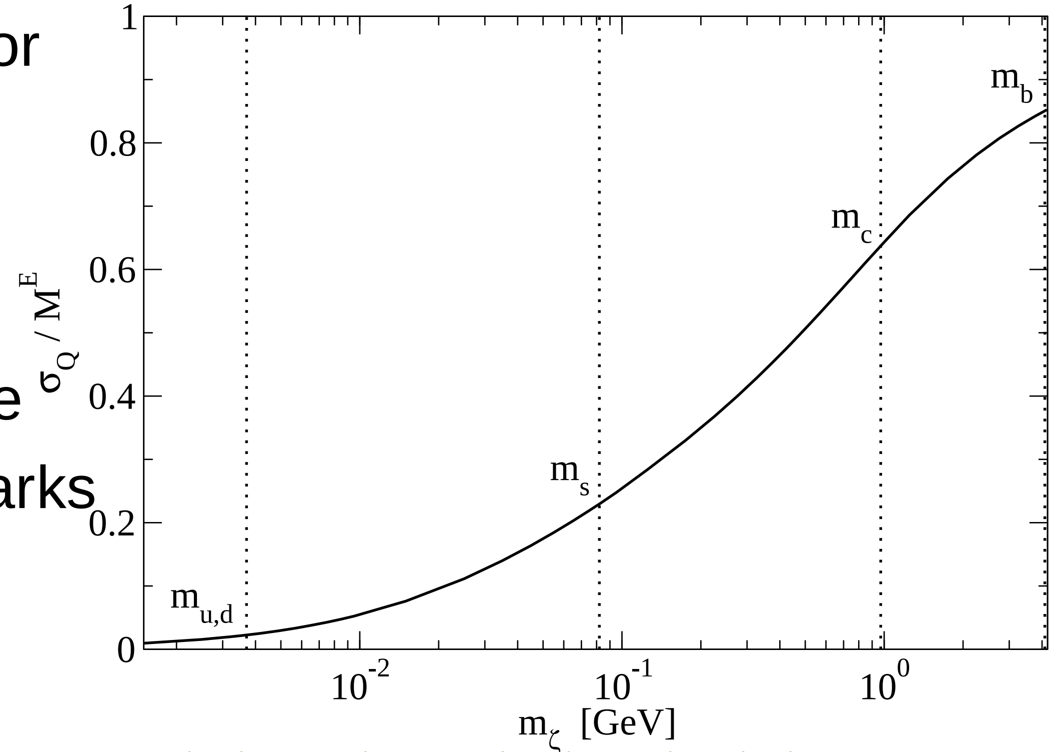
Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.



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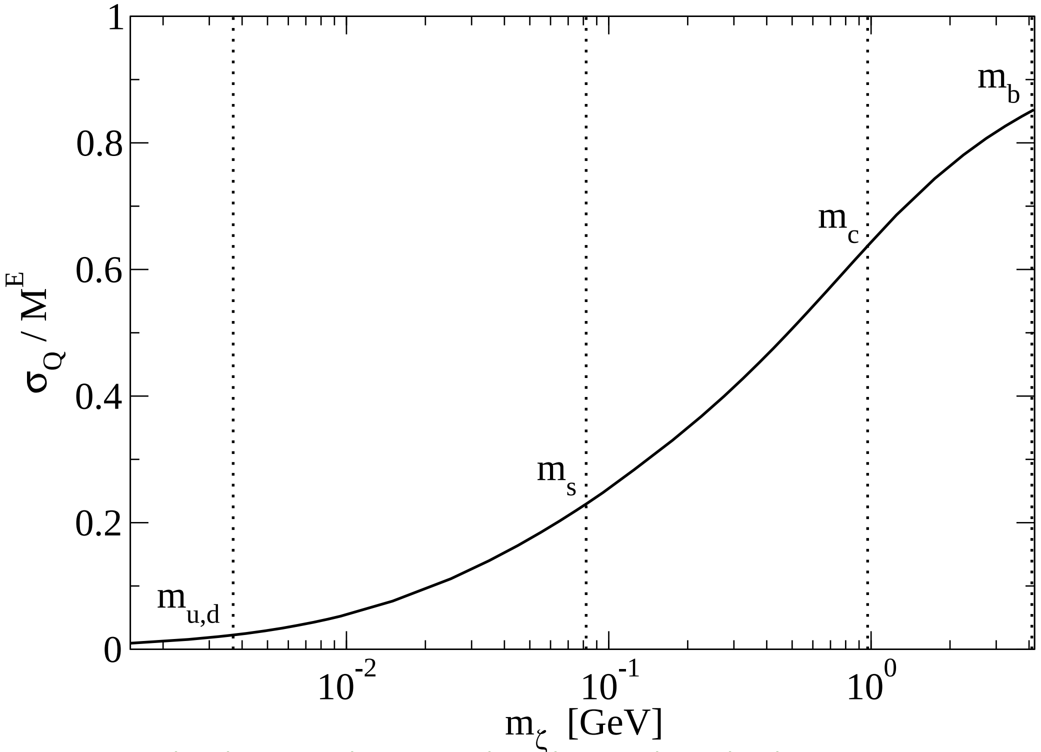
Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass



Hadrons

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

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  Comparison with experiment is
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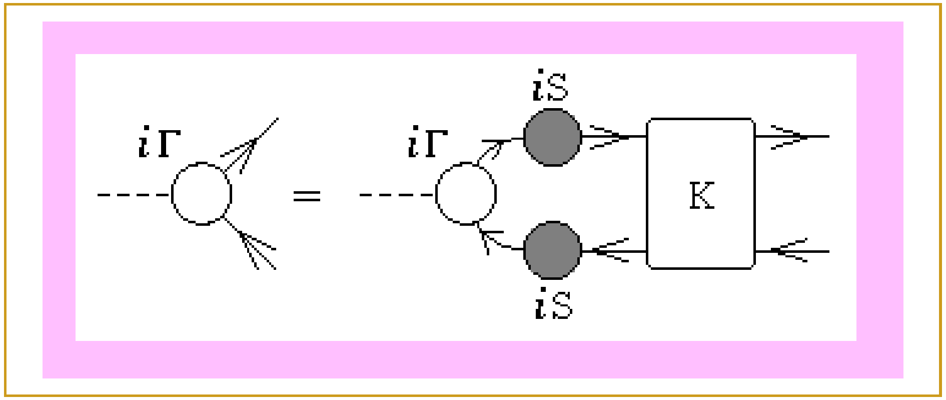
- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

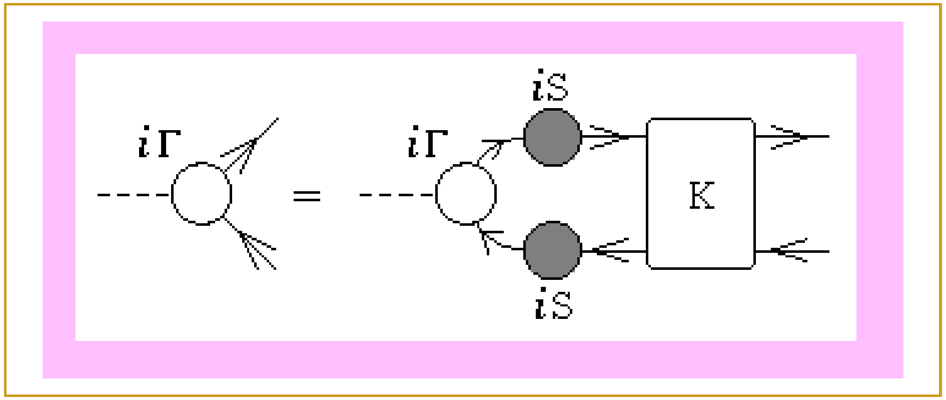
- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

- Without bound states, Comparison with experiment is impossible

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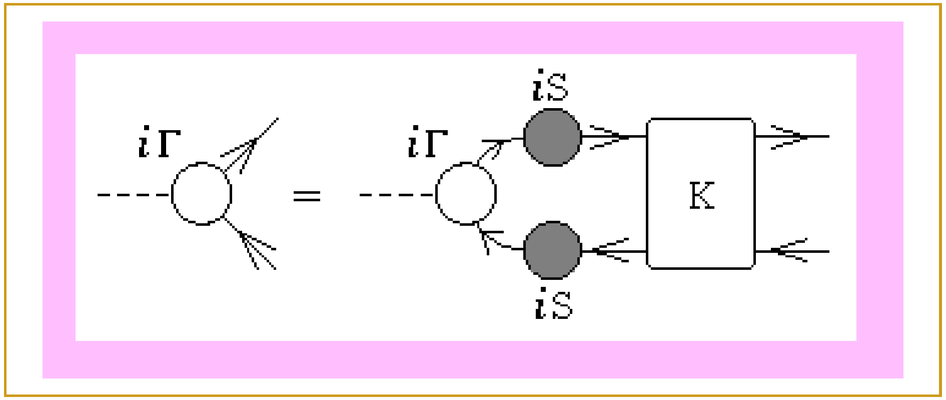

QFT Generalisation of Lippmann-Schwinger Equation.

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or

What is the light-quark Long-Range Potential?

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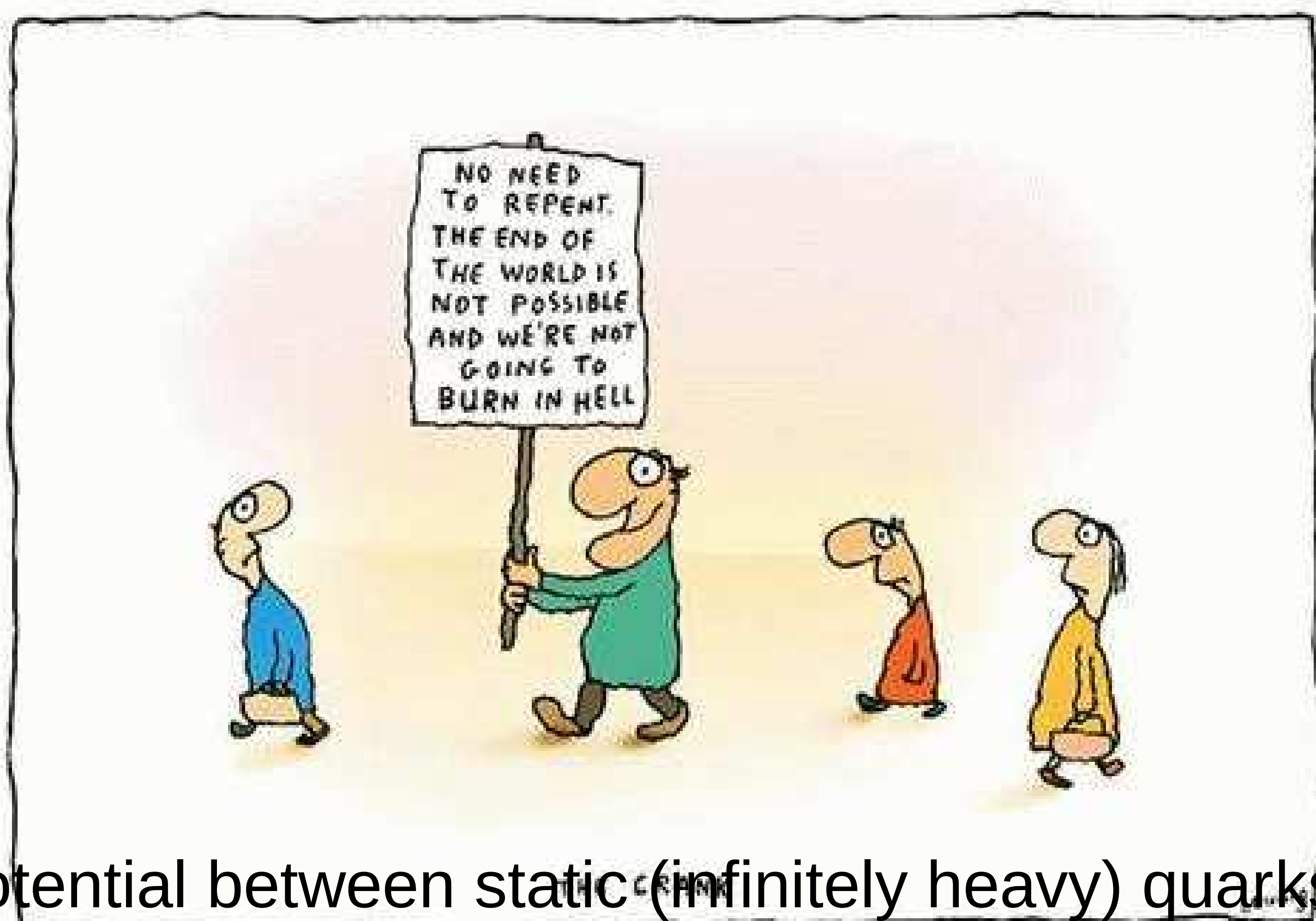

Potential between static (infinitely heavy) quarks measured in numerical simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\, \frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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Kernels very different

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Satisfies BSE — Satisfies DSE

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- Nontrivial constraint

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Satisfies BSE

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Kernels very different

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- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Radial Excitations
& Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\rm flavour}\left[M_{(\mu)}\left\{T^H, \left(T^H\right)^{\rm t}\right\}\right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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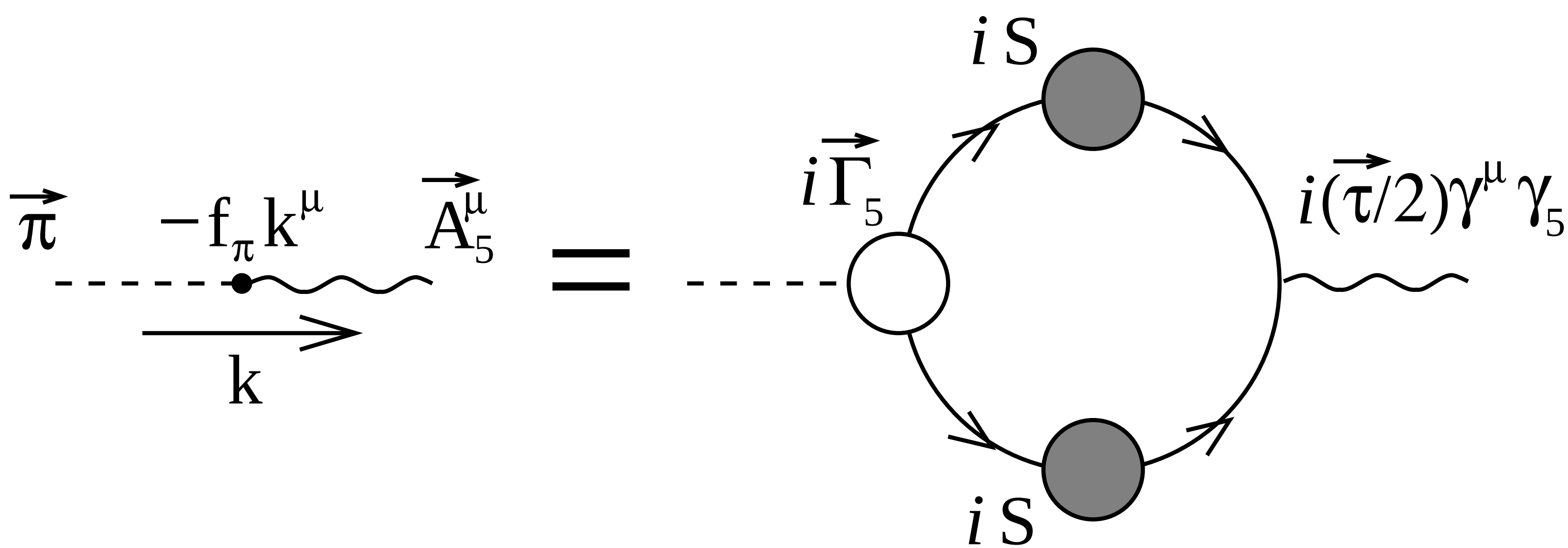
(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H\, m_H^2 = -\ \rho_\zeta^H\ \ \mathcal{M}_H}$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \gamma_\mu \boxed{\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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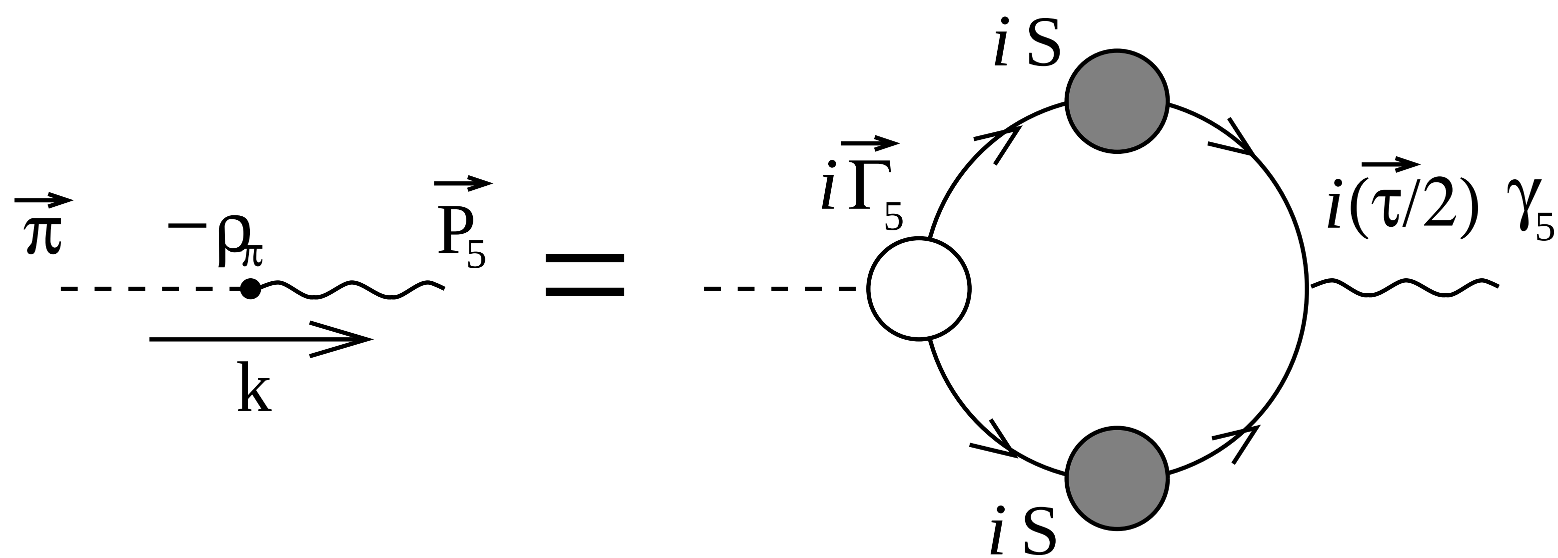

Radial Excitations & Chiral Symmetry

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q\rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q\rangle_\zeta^0}{(f_H^0)^2}\, m_q}$ . . . GMOR relation, a corollary

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Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} \, m_q}$ …GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

… QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$\boxed{f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H}$$

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Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
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- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

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  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit
- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

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- CLEO: $\tau \to \pi(1300) + \nu_\tau$
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  Diehl & Hiller
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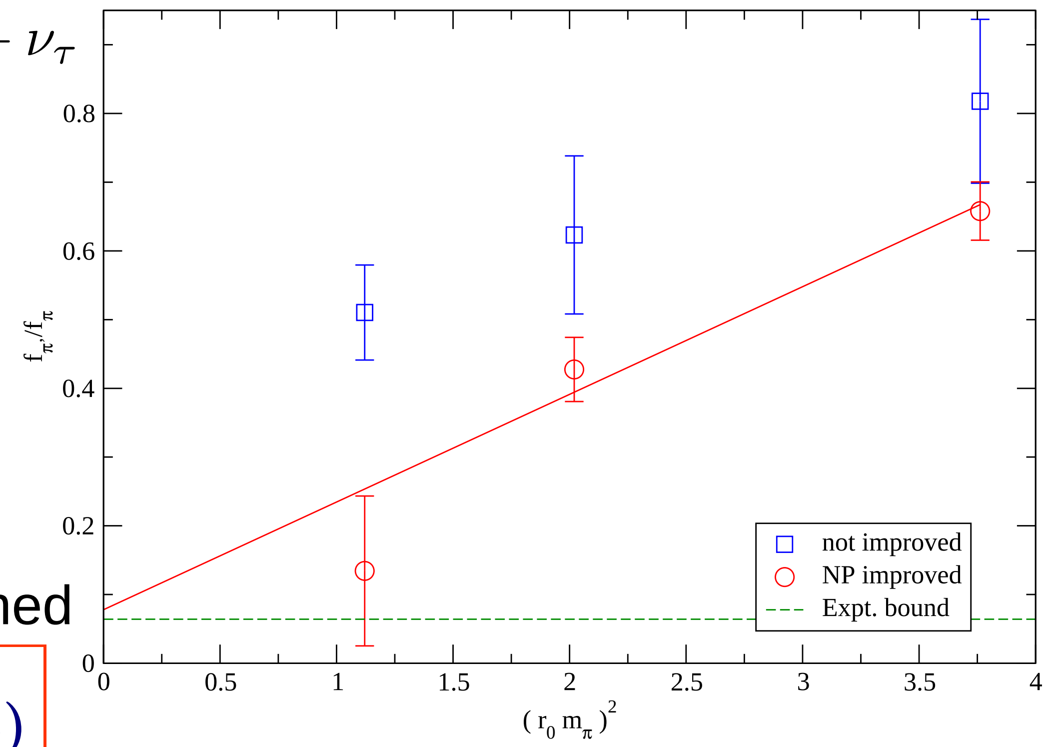
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1$ fm,
  two-flavour, unquenched

$$\boxed{\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)}$$



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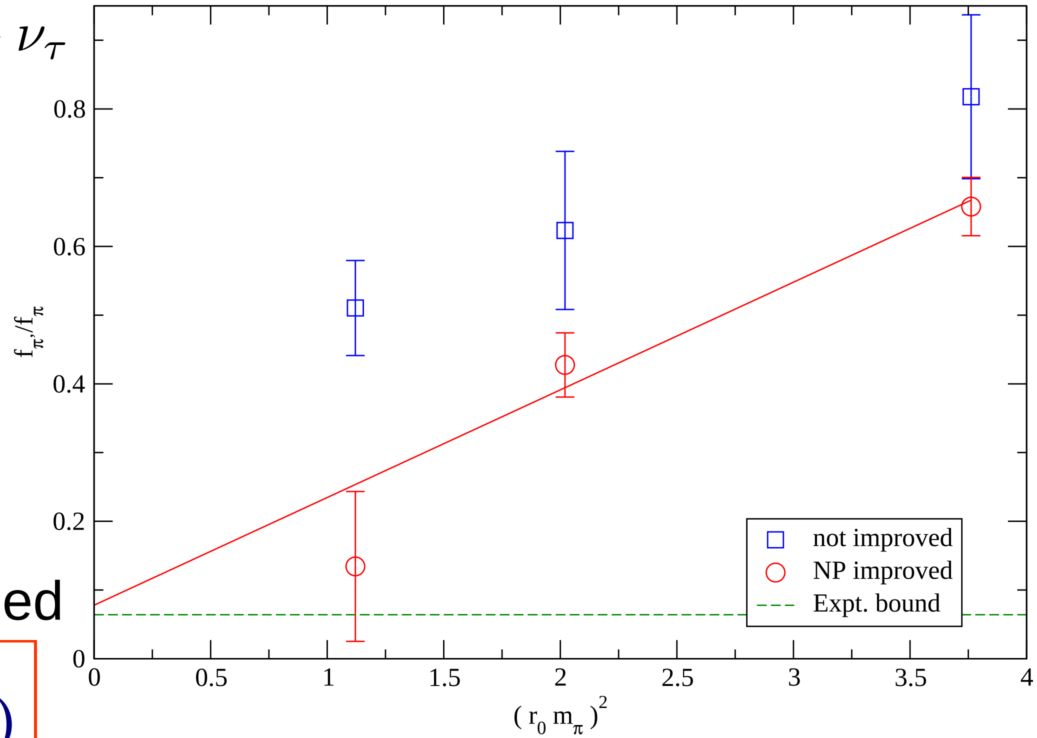

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

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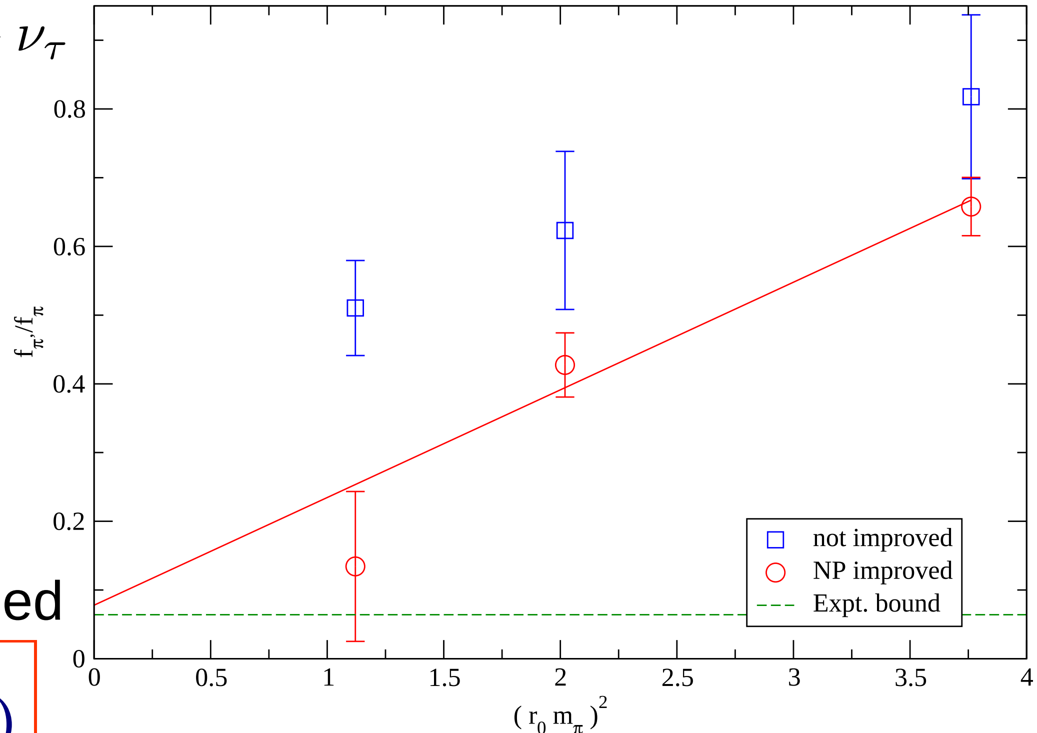
  $\Rightarrow f_{\pi_1} < 8.4$ MeV
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1$ fm,
  two-flavour, unquenched

$$\boxed{\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)}$$



- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion . . . $J = 0$
but . . .

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

First Contents Back Conclusion

Pion . . . $J = 0$ but . . .

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion . . . $J = 0$ but . . .

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right. \\ &\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

Pion ... $J = 0$ but ...

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- $J = 0 \ldots$ *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

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Introduce mixing angle θ_π such that

$$\chi_\pi \sim \cos\theta_\pi |L = 0\rangle + \sin\theta_\pi |L = 1\rangle$$

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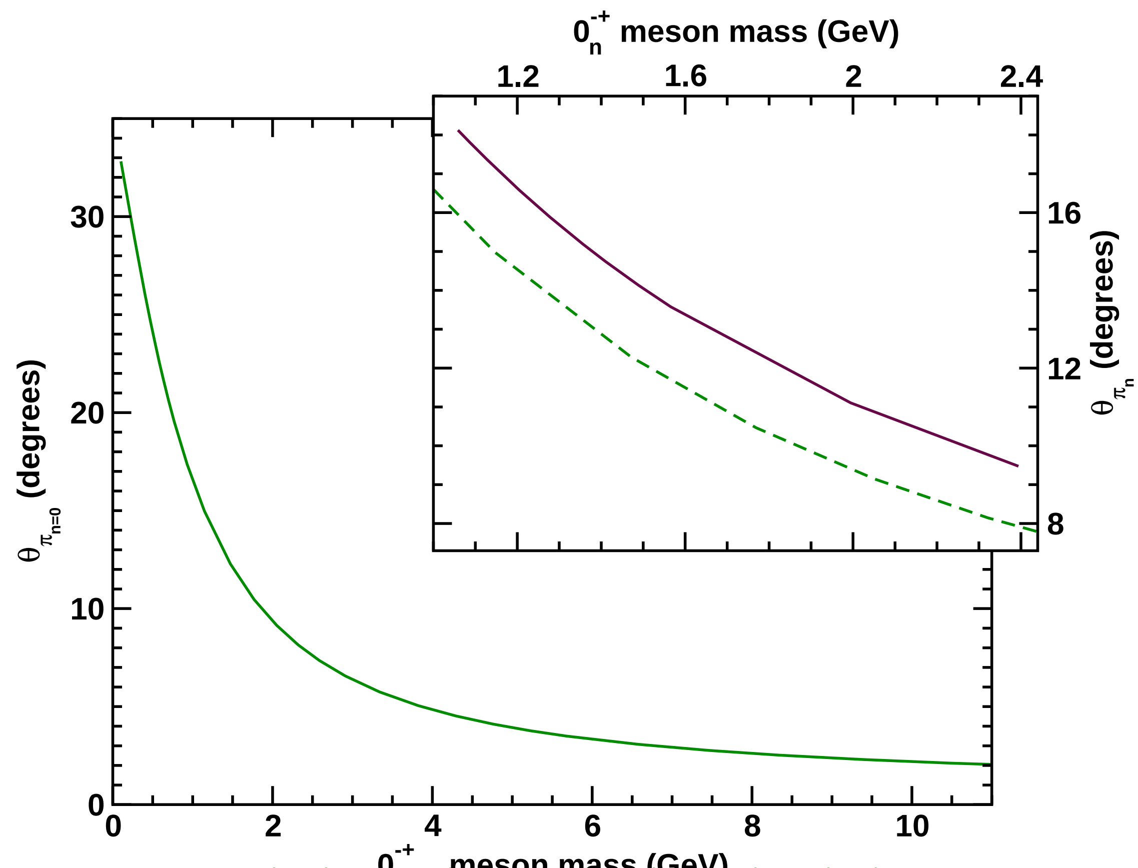

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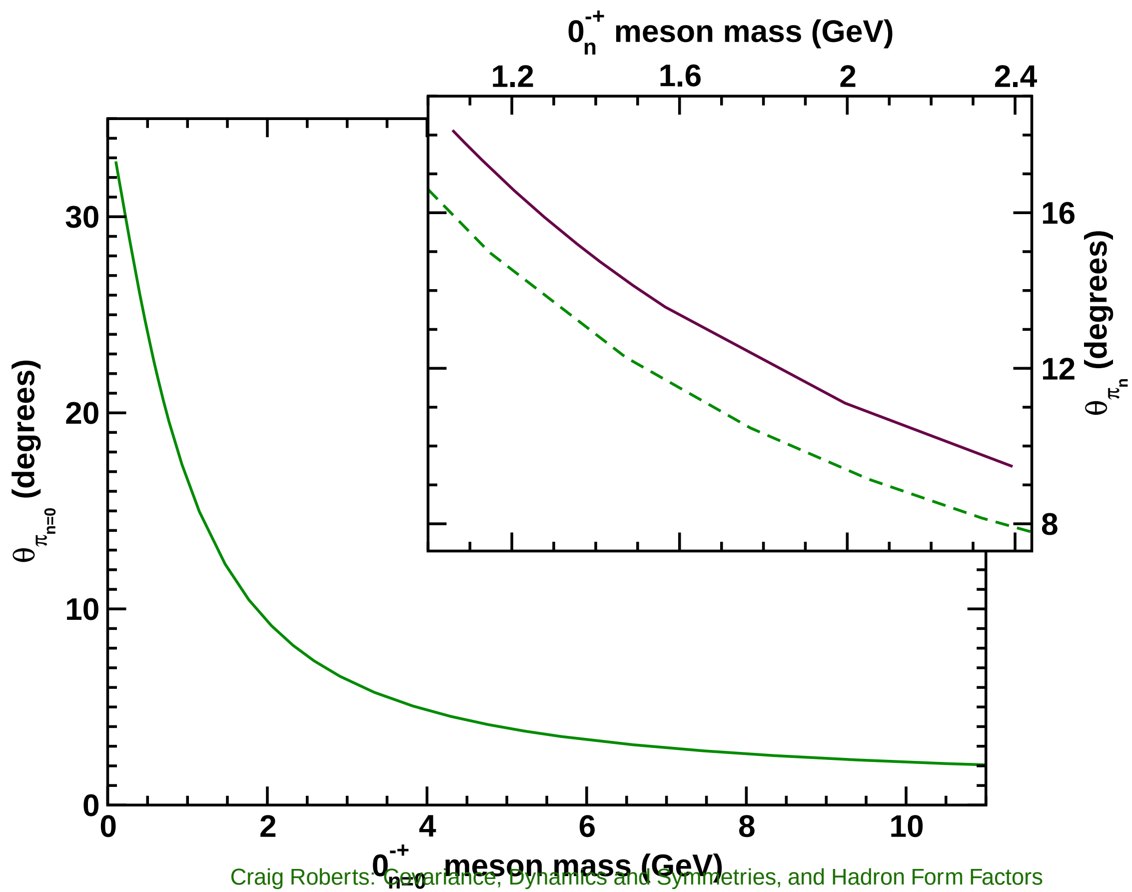
Introduce mixing angle θ_π such that

$$\chi_\pi \sim \cos\theta_\pi |L = 0\rangle + \sin\theta_\pi |L = 1\rangle$$

L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.

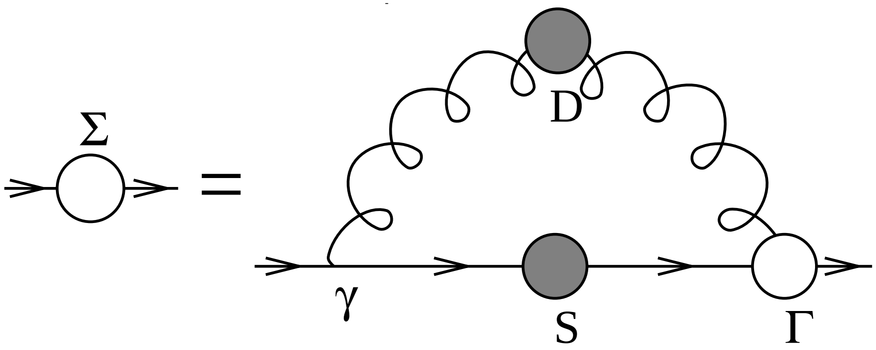


Pion Form Factor

Procedure Now Straightforward

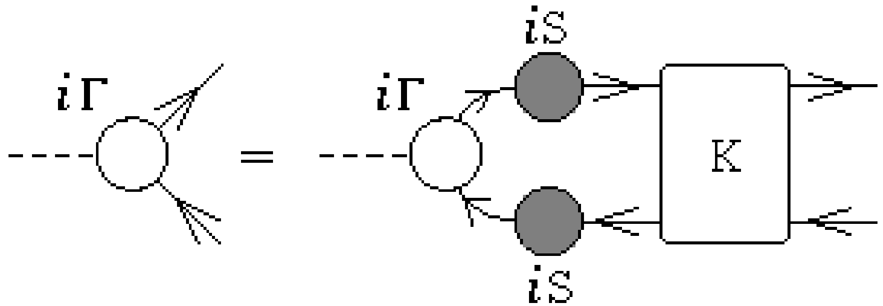
First | Contents | Back | Conclusion

- Solve Gap Equation
  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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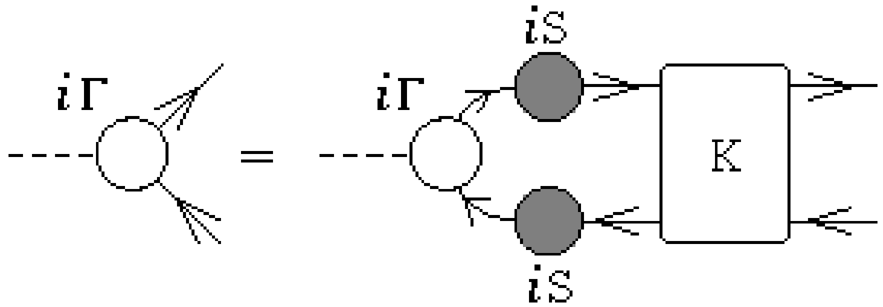
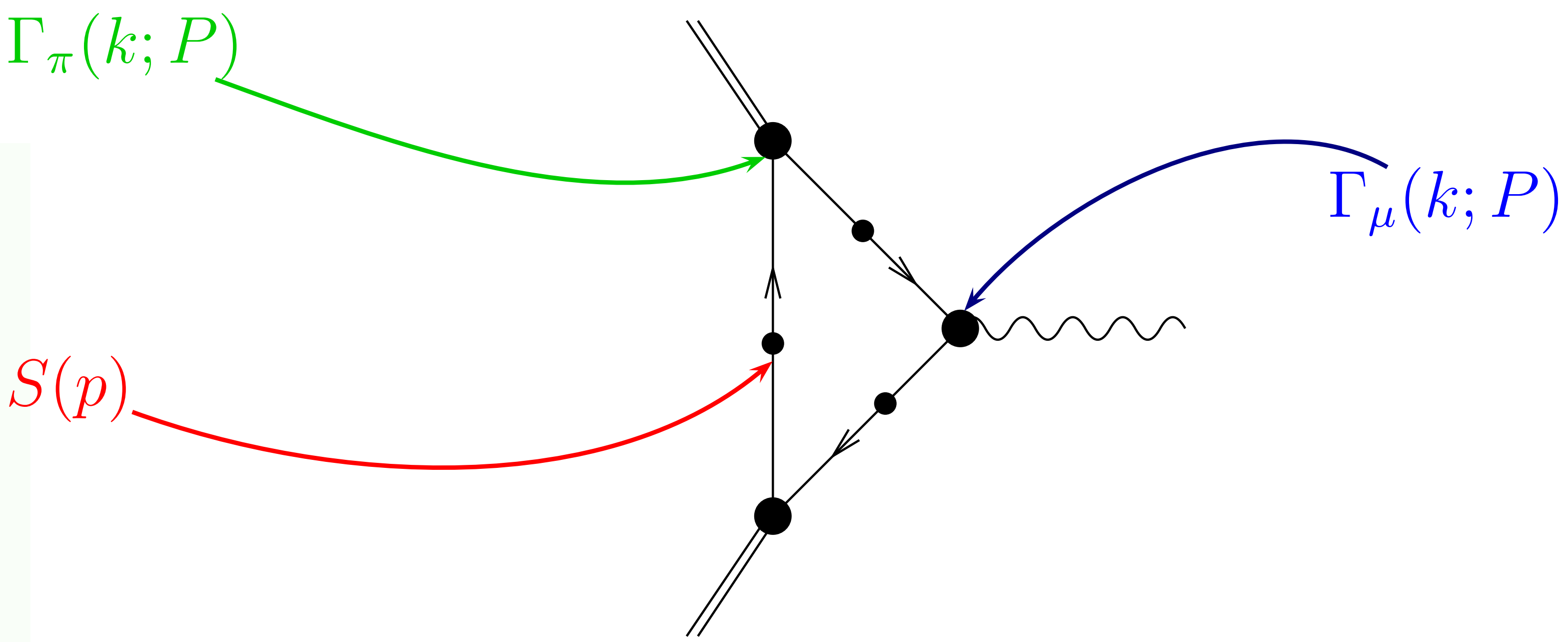


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, Γ_μ

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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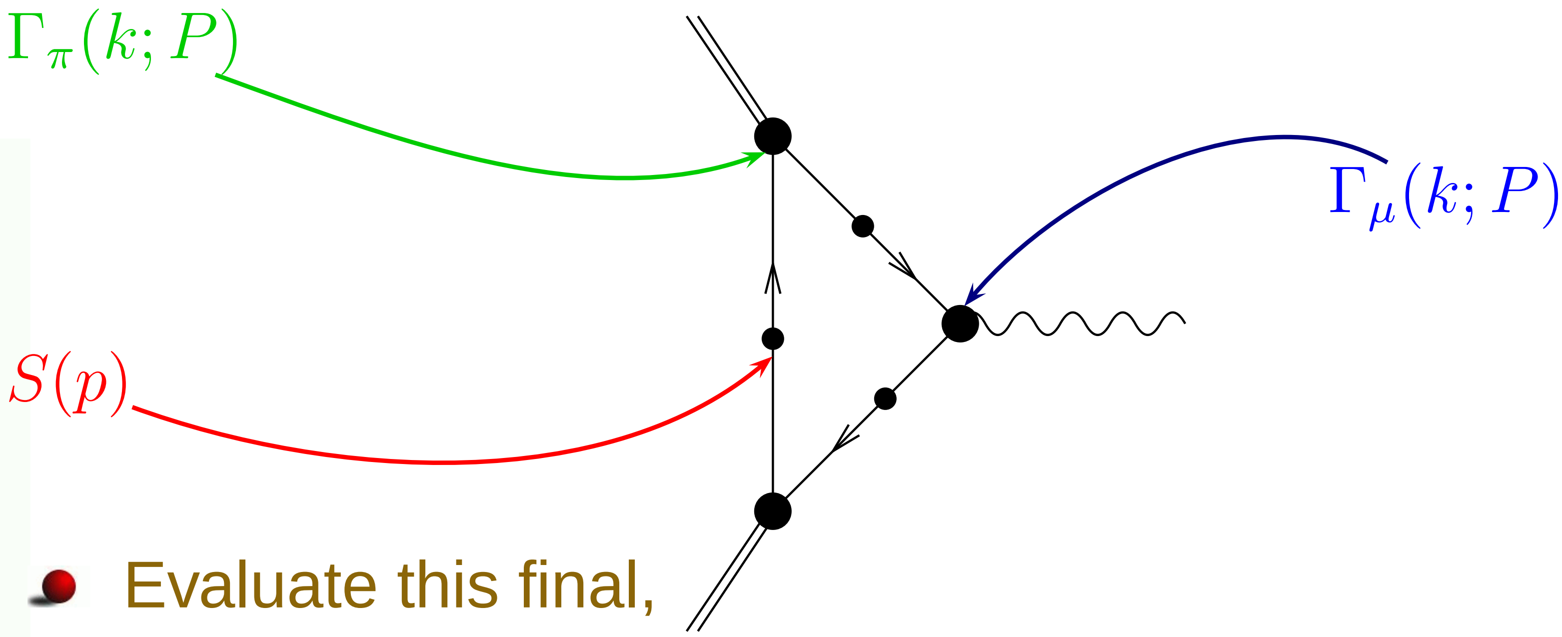
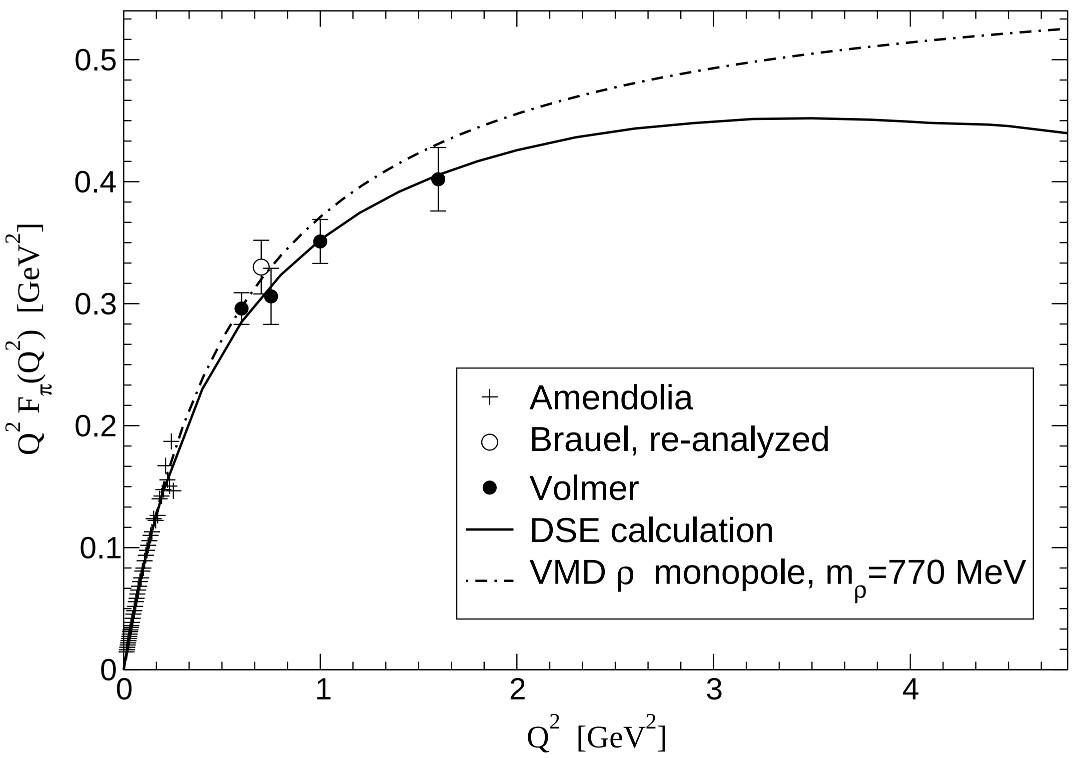


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

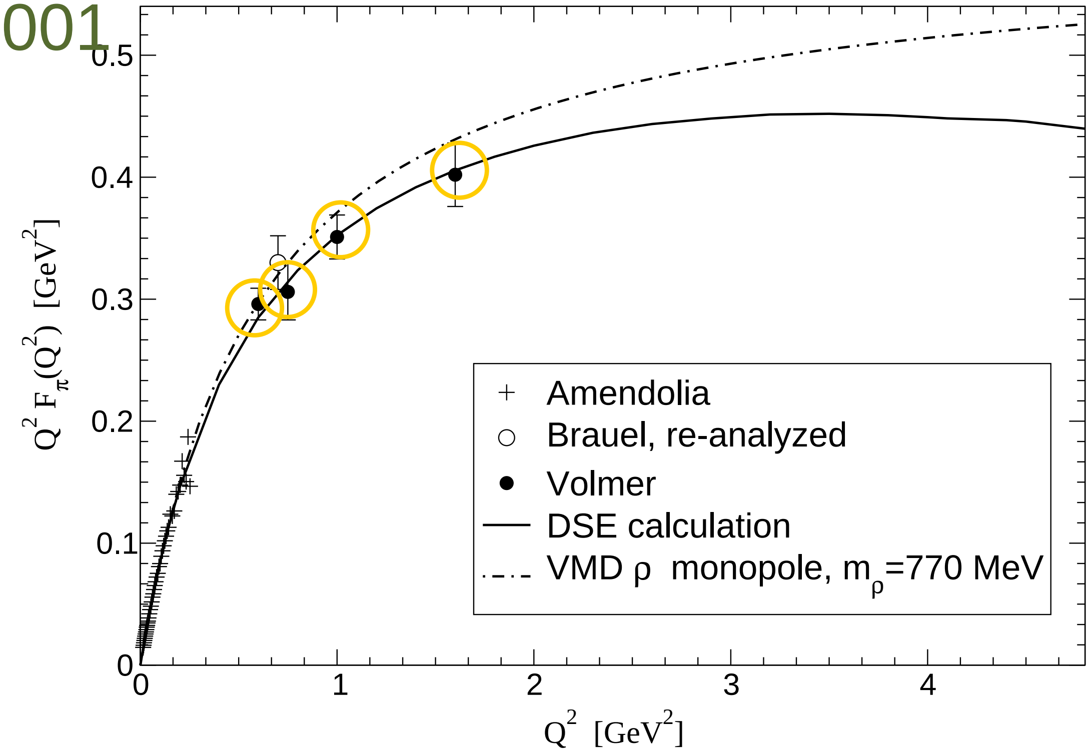
Calculation published in 1999; No Parameters Varied



Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001



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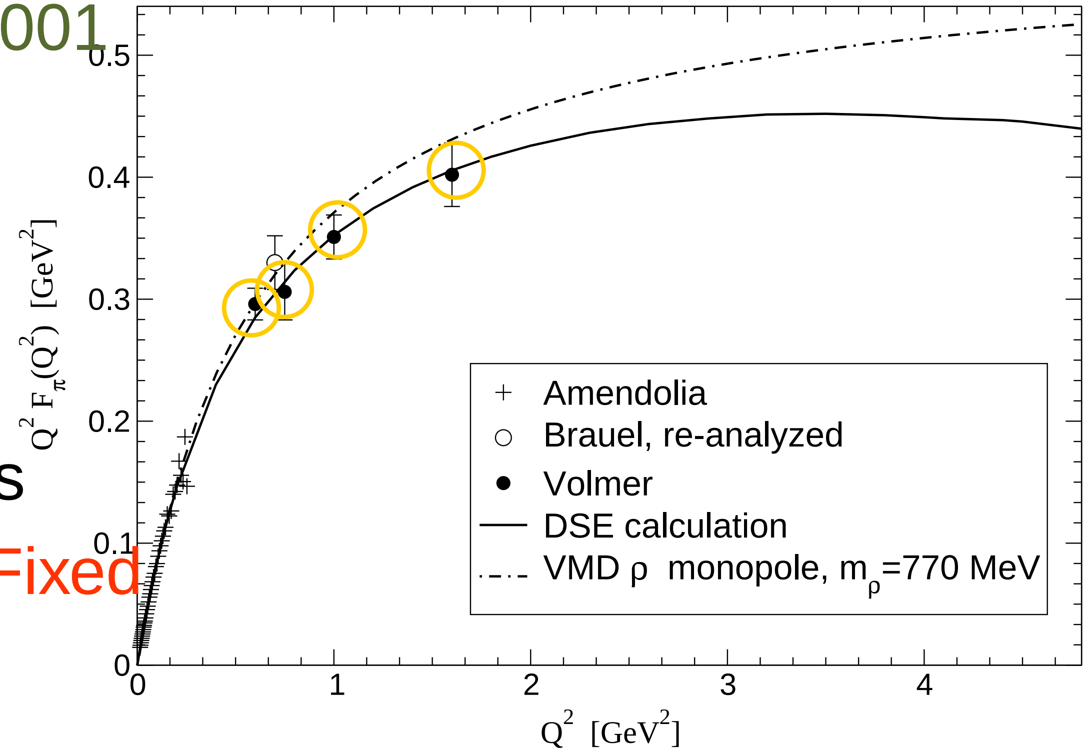
Data published in 2001



Many subsequent successful applications

. . Again, parameters Fixed

Notably $\pi\ \pi$ Scattering

Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

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Dimensionless product: $r_\pi \, f_\pi$

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- Improved rainbow-ladder interaction

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Dimensionless product: $r_\pi \, f_\pi$

- Improved rainbow-ladder interaction
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- Great strides towards placing nucleon studies on same footing as mesons

Dimensionless product: $r_\pi \, f_\pi$

- Improved rainbow-ladder interaction
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- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

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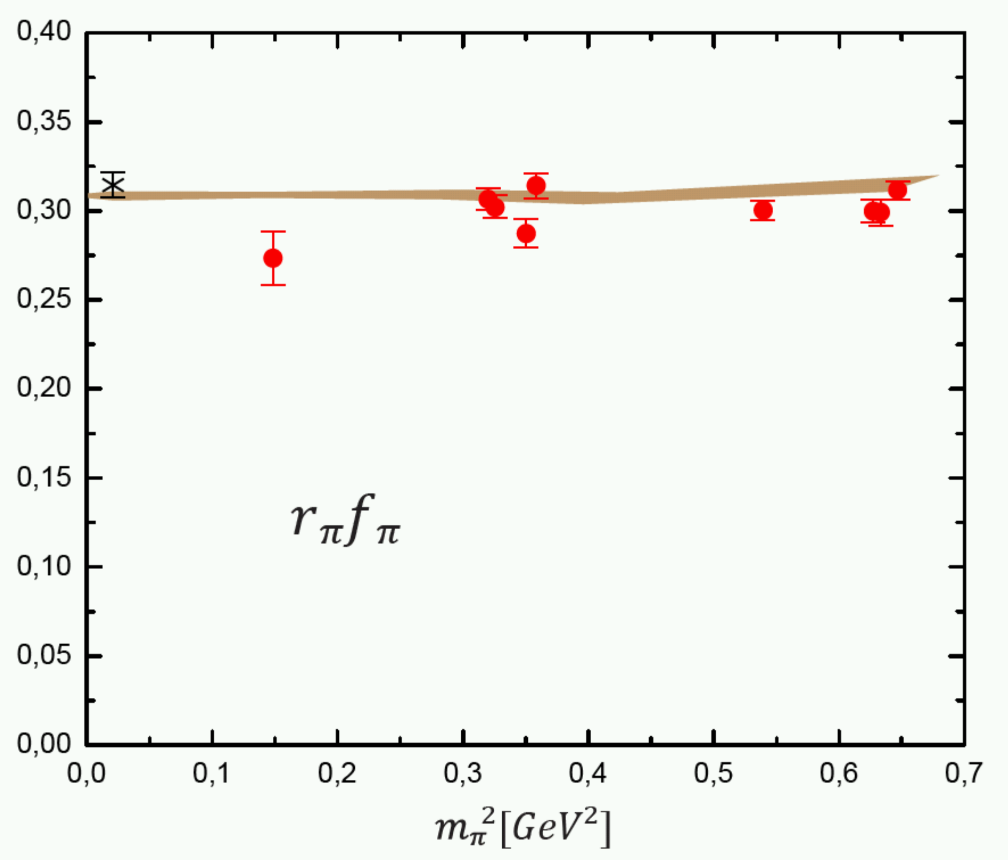
- DSE prediction



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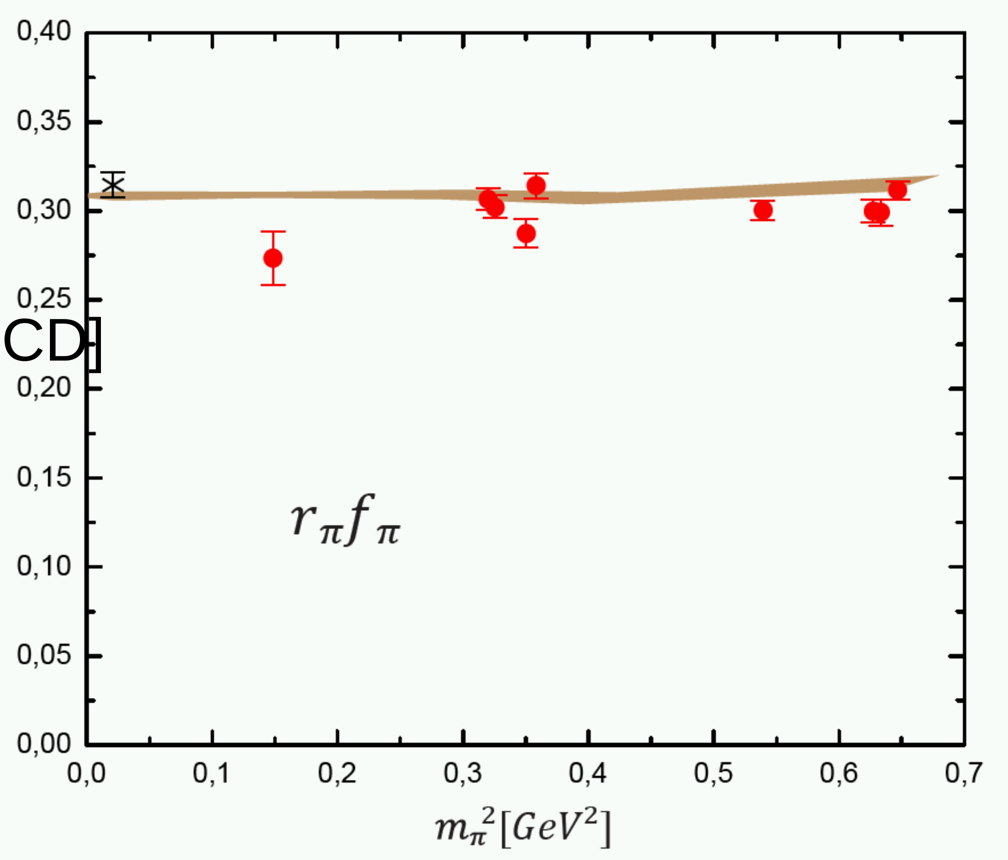

- DSE prediction
- Lattice results
  – James Zanotti [UK QCD]

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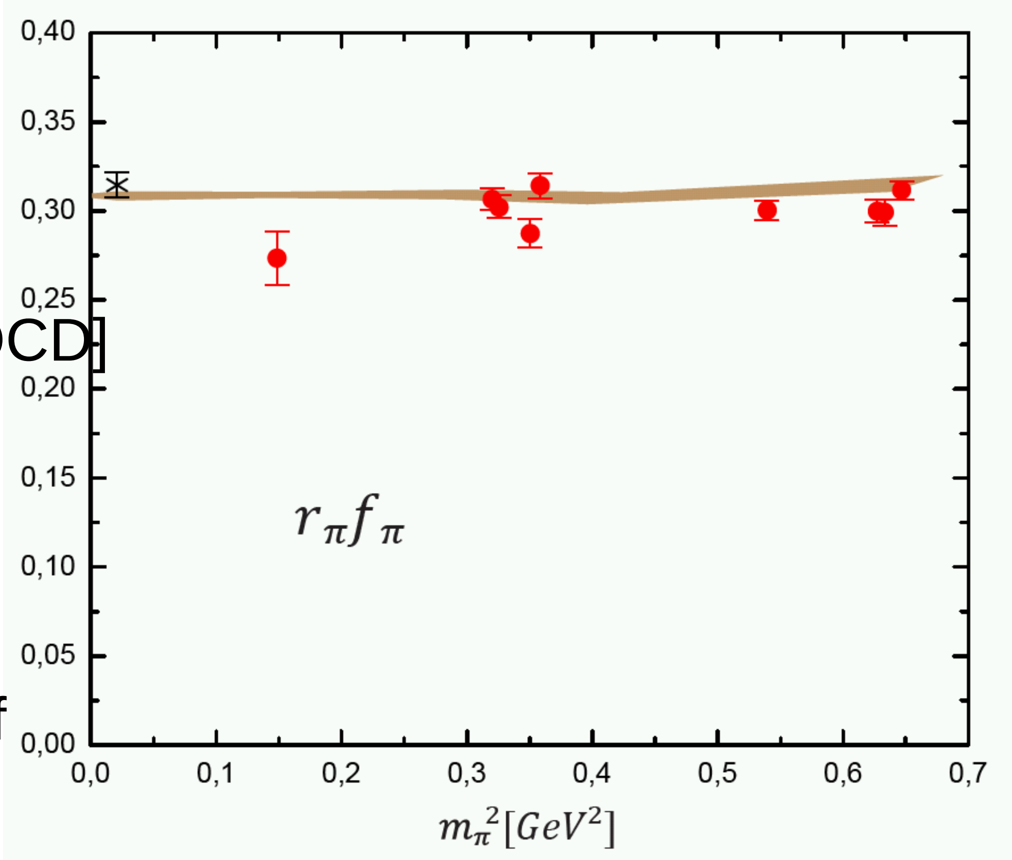

- DSE prediction
- Lattice results

  – James Zanotti [UK QCD]
- Fascinating result:

  DSE and Lattice

  – Experimental value obtains independent of current-quark mass.

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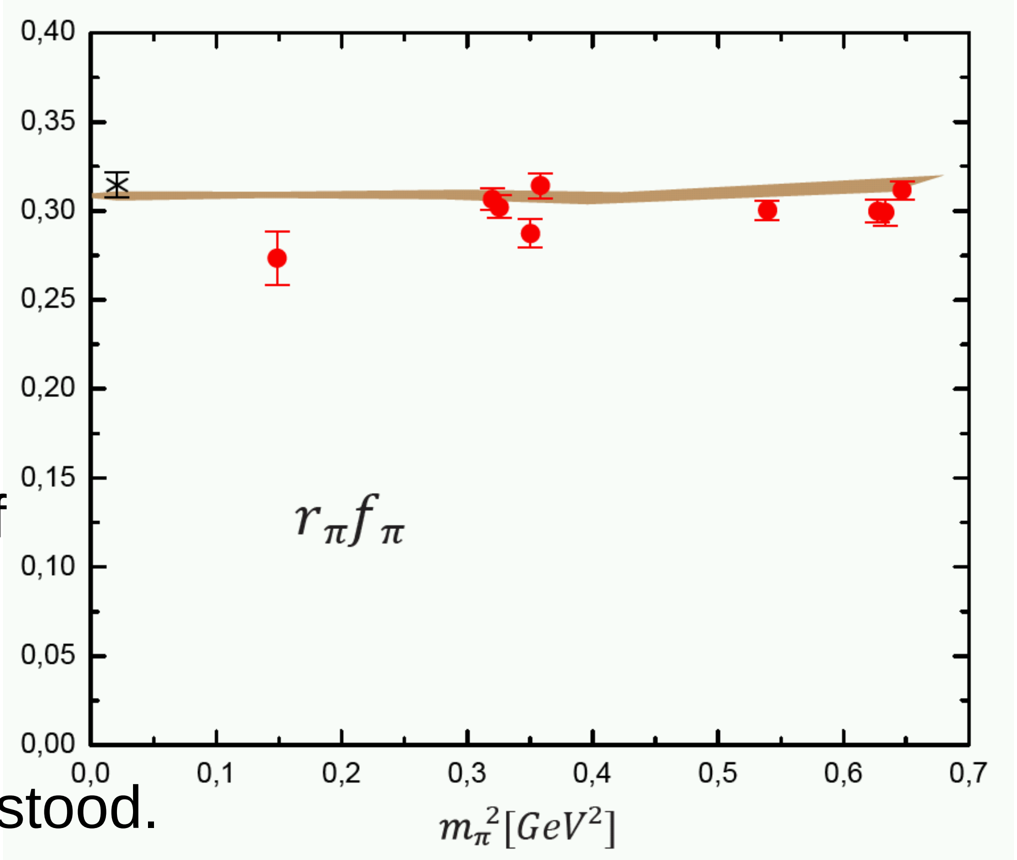

- DSE prediction
- Fascinating result: DSE and Lattice – Experimental value obtains independent of current-quark mass.
- Potentially useful but must first be understood.

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Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
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- $\{\mathcal{F}^a \,|\, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[\{\mathcal{F}^a, \mathcal{M}\}\mathcal{F}^b\right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses

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- The final term in the second line expresses the non-Abelian axial anomaly.

Charge Neutral Pseudoscalar Mesons

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P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
\end{aligned}
$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\,\mathcal{Q}(0)\,\bar{q}(y)\rangle$

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- $\mathcal{Q}(x) = i\dfrac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.

Charge Neutral Pseudoscalar Mesons

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  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

Charge Neutral Pseudoscalar Mesons

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  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\ -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

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  . . . The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of $U_A(1)$ problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

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Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

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Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
$$\left. + \gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_{\mathcal{A}}(k;P)\right]$$

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Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars
Argonne NATIONAL LABORATORY

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Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f_{\eta'}^0 E_{BS}(k;0) &= 2 B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F_R^0(k;0) + 2 f_{\eta'}^0 F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G_R^0(k;0) + 2 f_{\eta'}^0 G_{BS}(k;0) &= 2 A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H_R^0(k;0) + 2 f_{\eta'}^0 H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
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A_0, B_0 characterise gap equation's chiral limit solution.

Charge Neutral Pseudoscalar Mesons

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2 f^0_{\eta'} E_{BS}(k;0) &= 2 B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0),\\
F^0_R(k;0) + 2 f^0_{\eta'} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0),\\
G^0_R(k;0) + 2 f^0_{\eta'} G_{BS}(k;0) &= 2 A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0),\\
H^0_R(k;0) + 2 f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$ is necessary and sufficient condition for absence of massless $\boldsymbol{\eta'}$ bound-state.

Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

Bhagwat, Chang, Liu, Roberts, Tandy
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- Further highlighted . . . proved

$$\begin{aligned} \langle \bar{q}q \rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\mathrm{CD}} \int_q^\Lambda S^0(q,\zeta) \\ &= N_f \int d^4x\, \langle \bar{q}(x) i\gamma_5 q(x) \mathcal{Q}(0)\rangle^0 . \end{aligned}$$

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Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

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- Employed in an analysis of pseudoscalar- and vector-meson bound-states

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Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .
    $\lesssim 75\,\%$ current-quark mass-difference

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New Challenges

New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

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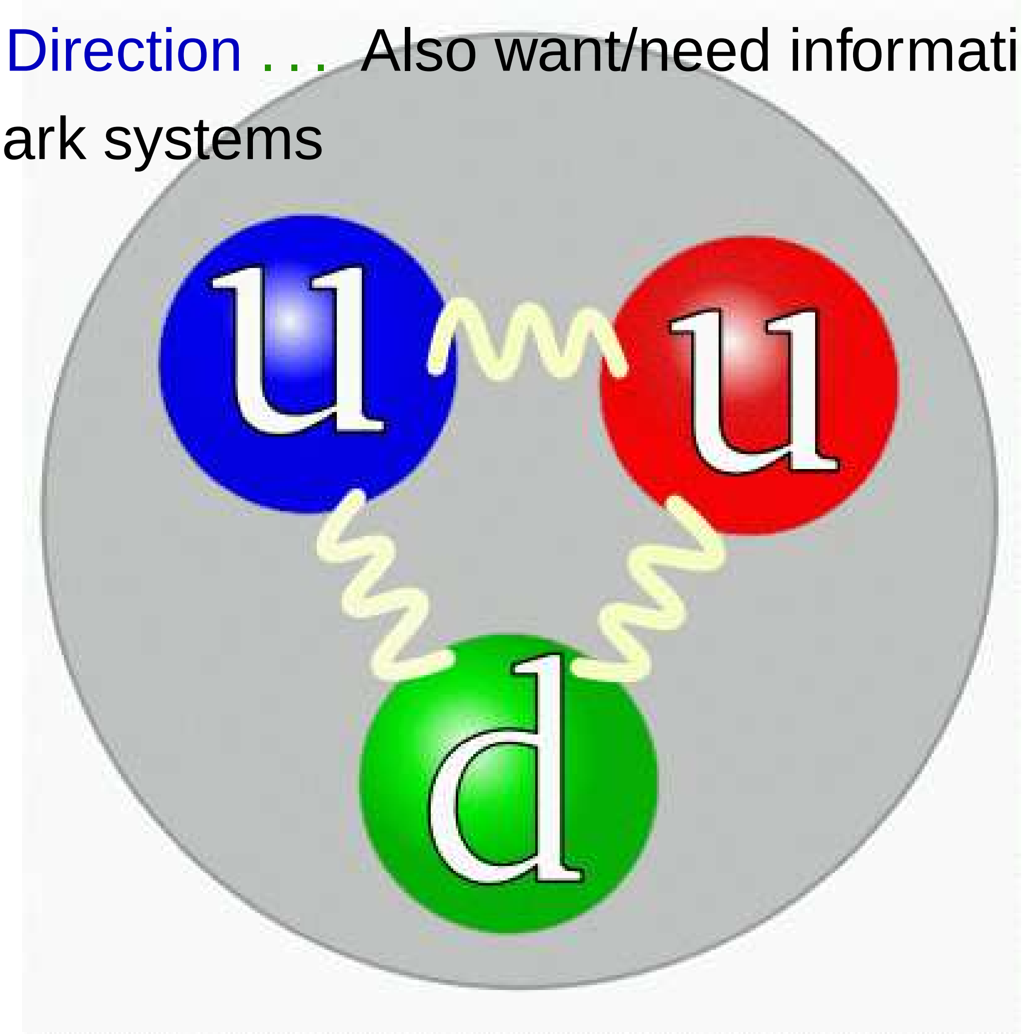
New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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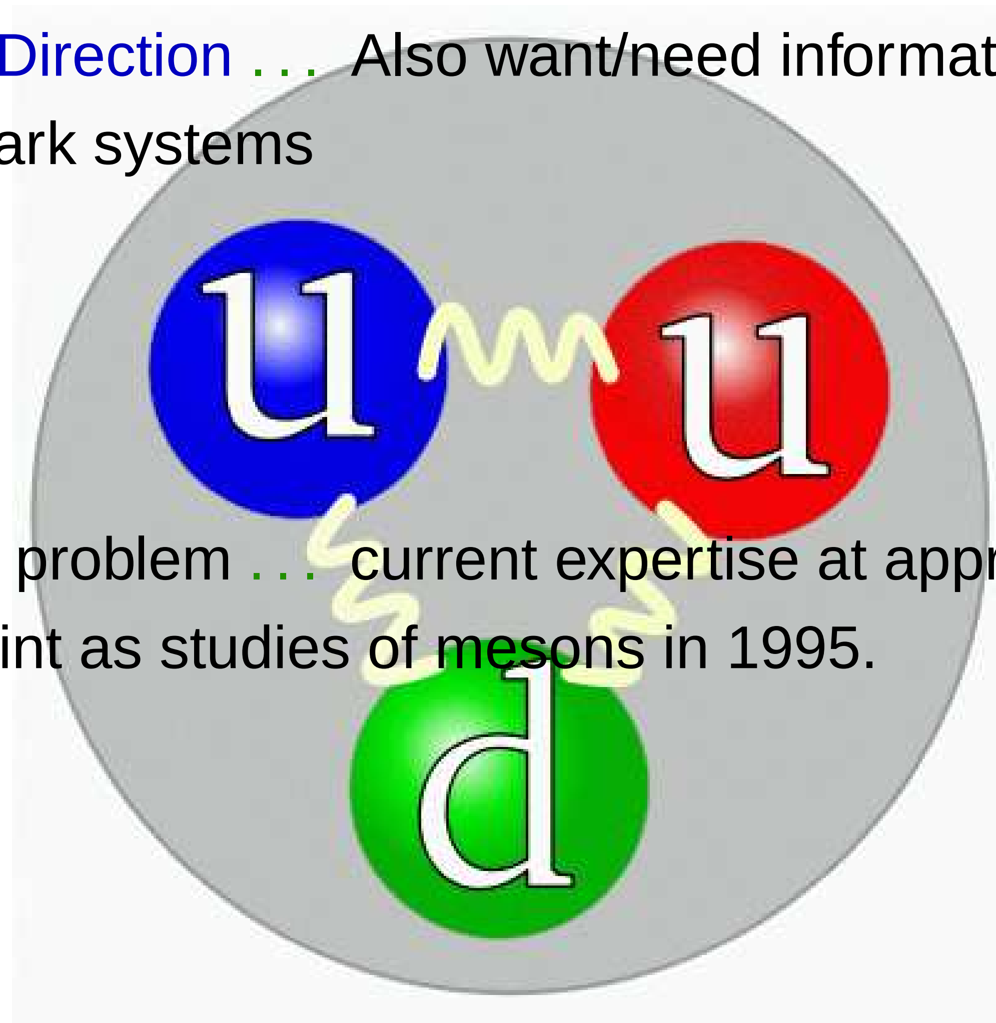
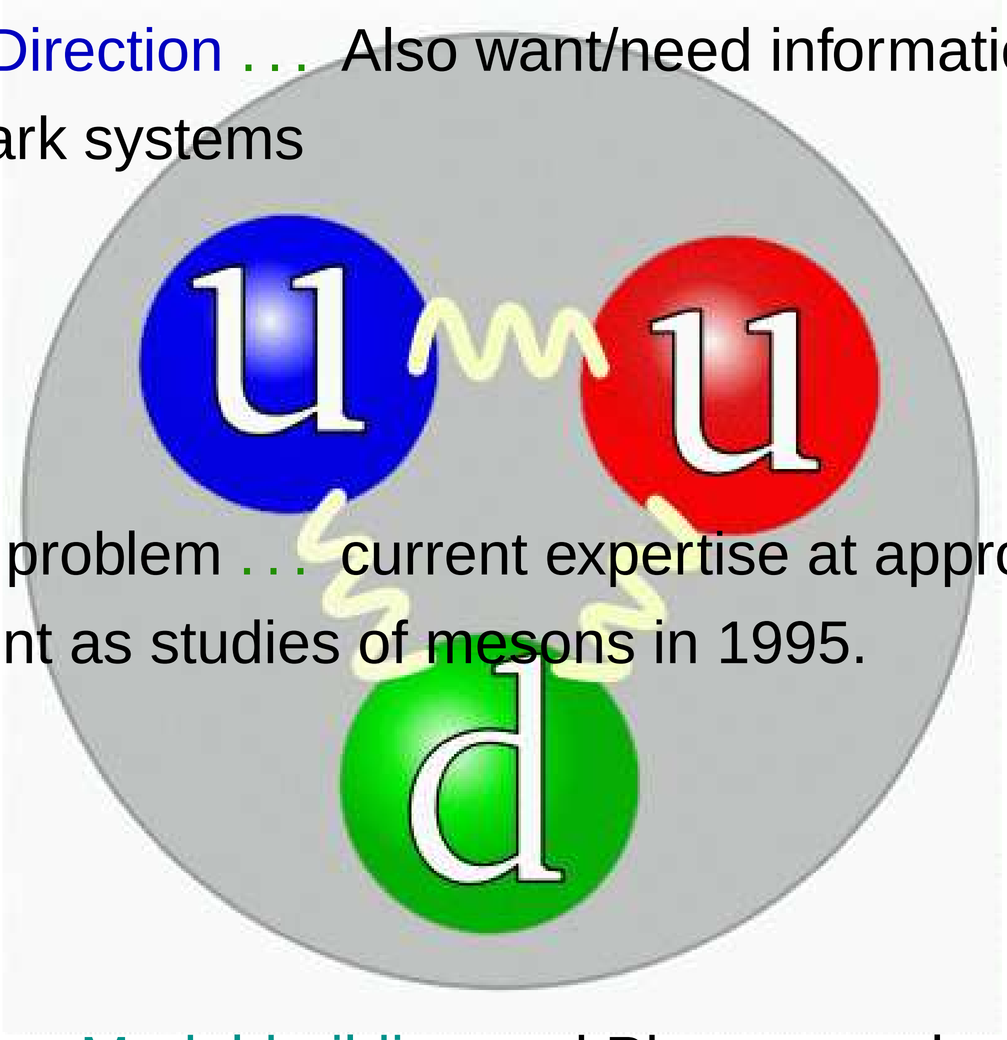


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

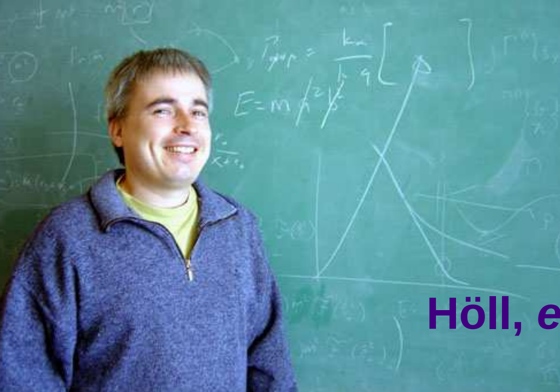
New Challenges

- Another Direction . . . Also want/need information about three-quark systems



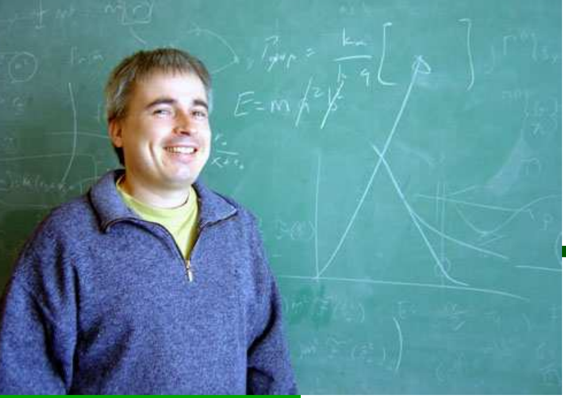
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033



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- Excellent mass spectrum (octet and decuplet)

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$$\left(\frac{1}{N_H}\sum_H \frac{[M_H^{\rm exp}-M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2}\right)^{1/2} = 2\%$$

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

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Faddeev equation

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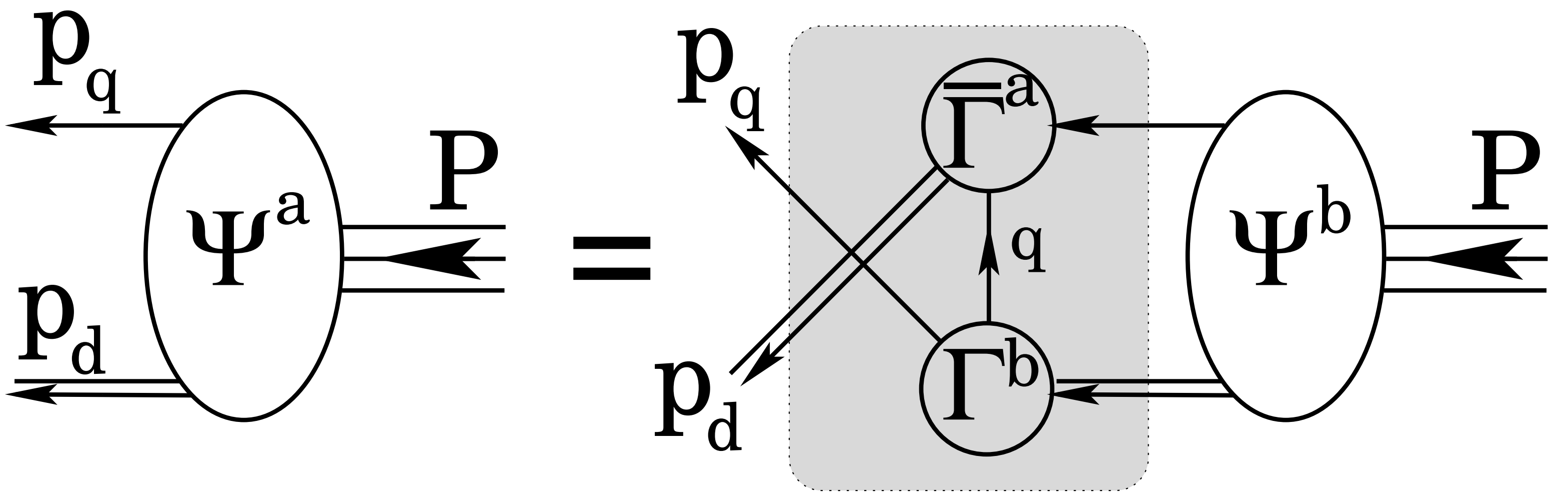

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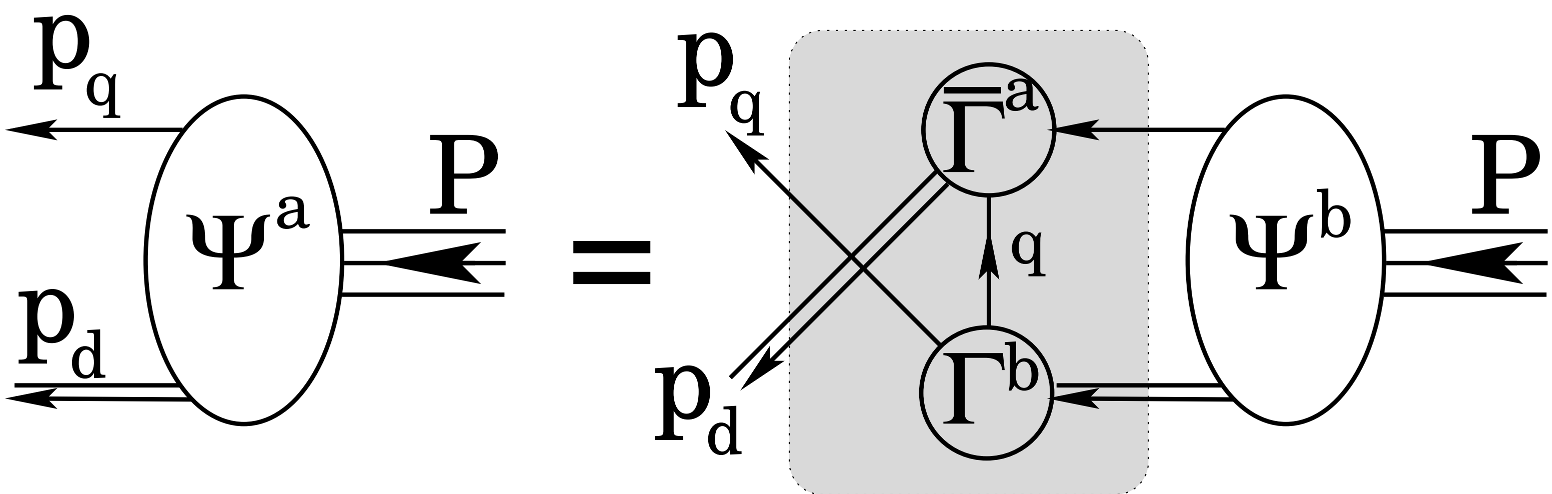
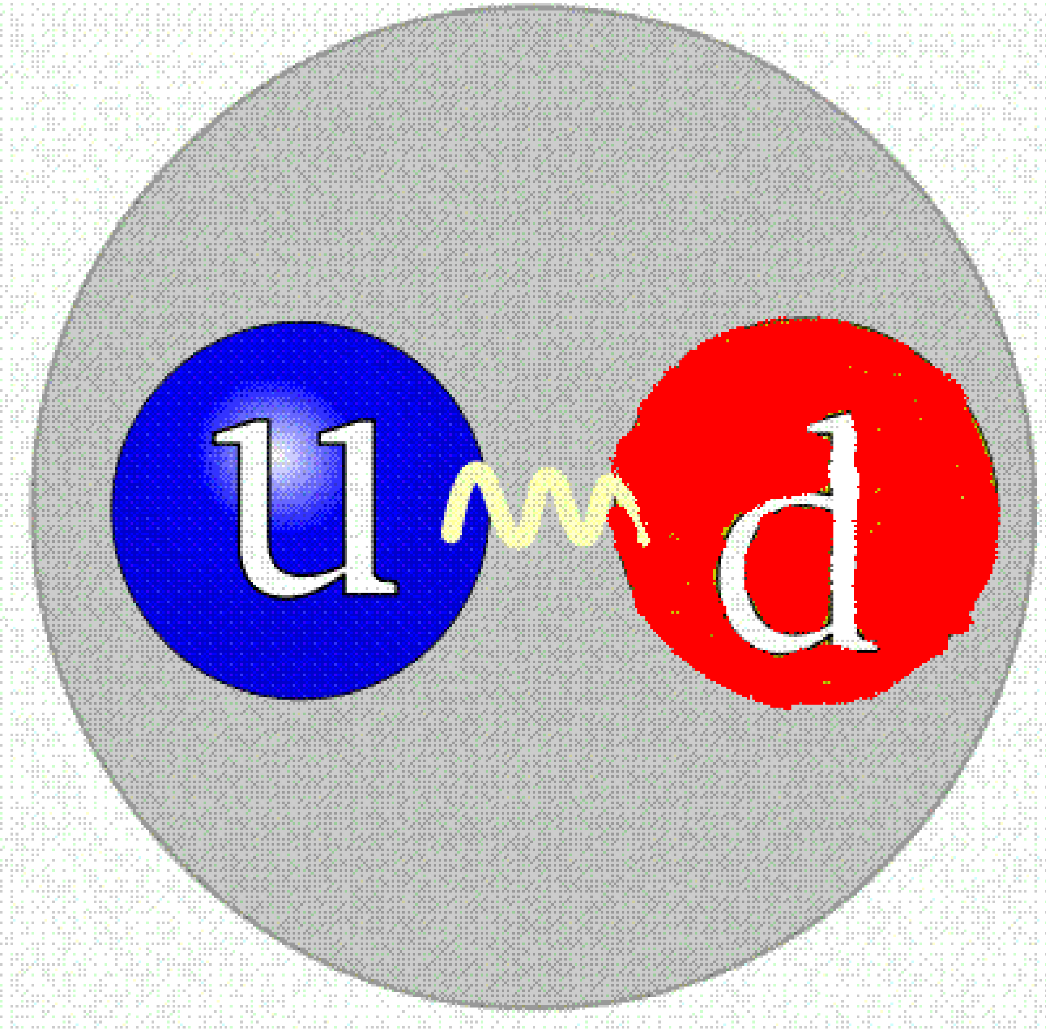


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

Diquark correlations



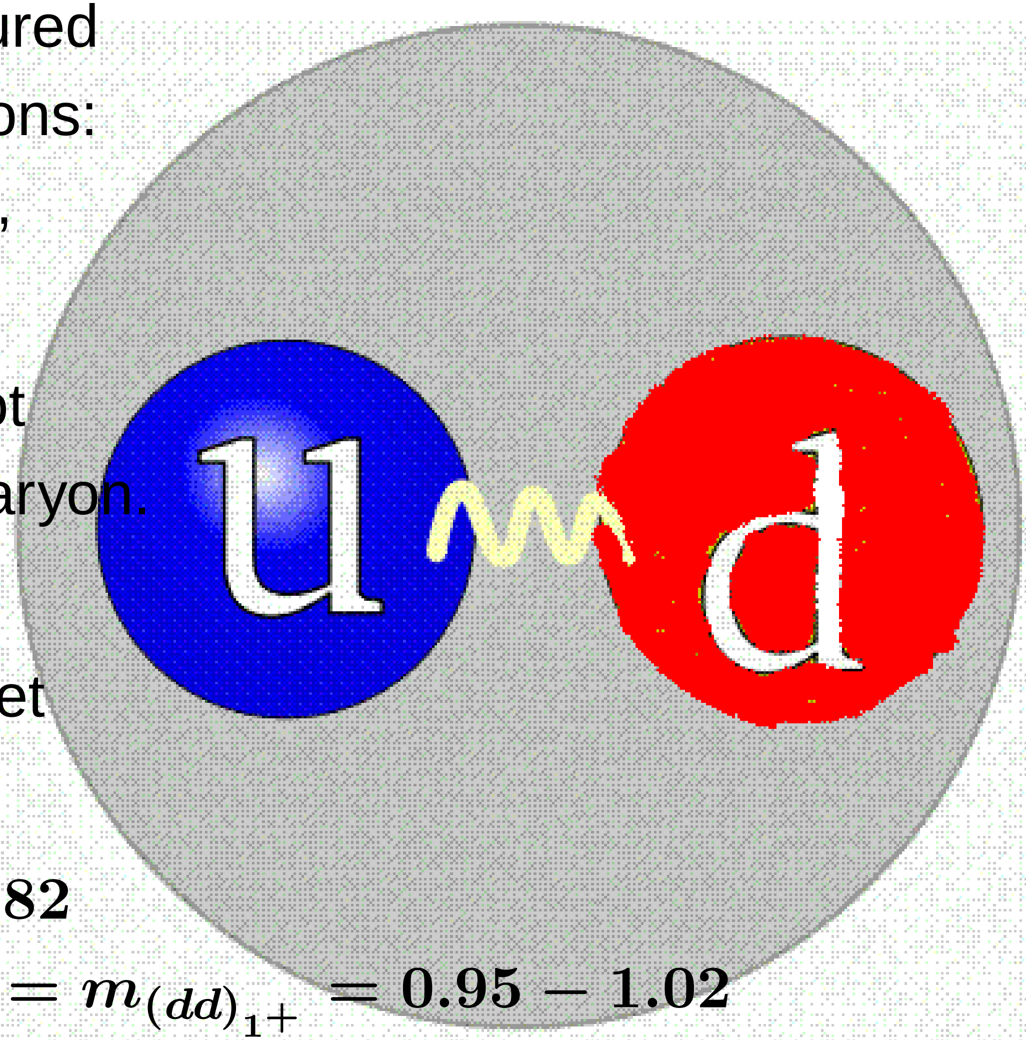
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

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Pions and Form Factors

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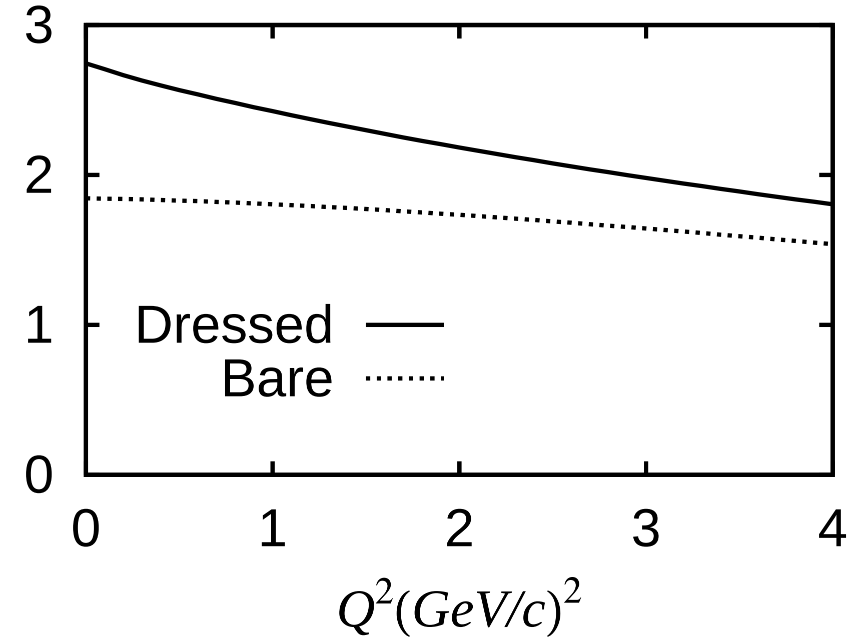
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G^*_M(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



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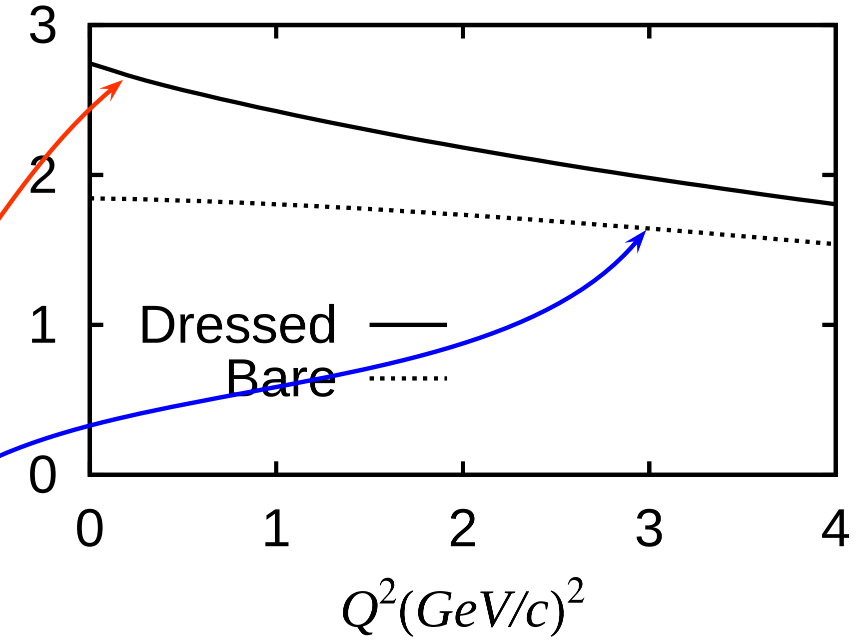

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 > 2 - 3\,\text{GeV}^2$

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for
Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0^+}	m_{1^+}	ω_{0^+}	ω_{1^+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.80	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1^+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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Nucleon-Photon Vertex

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M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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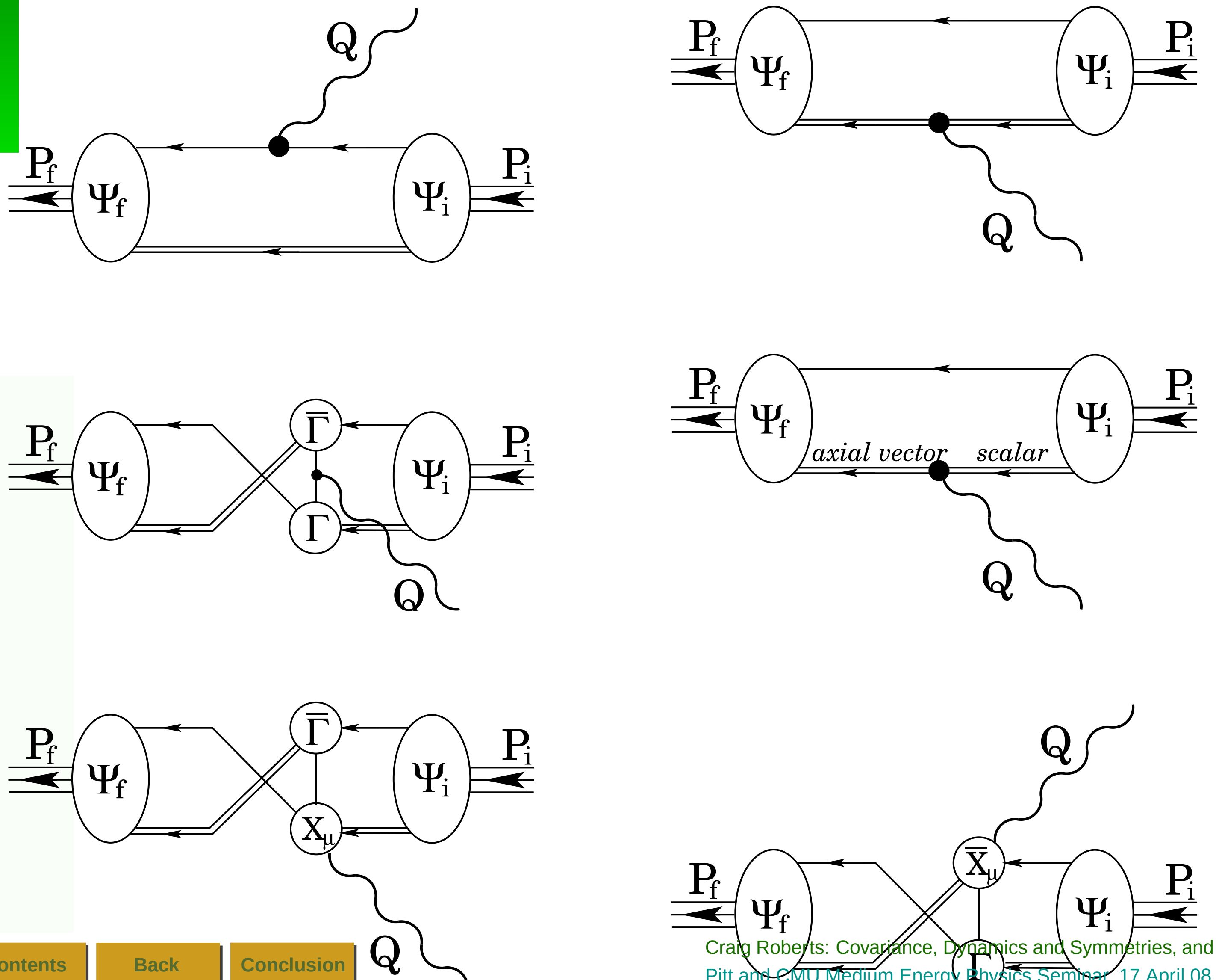
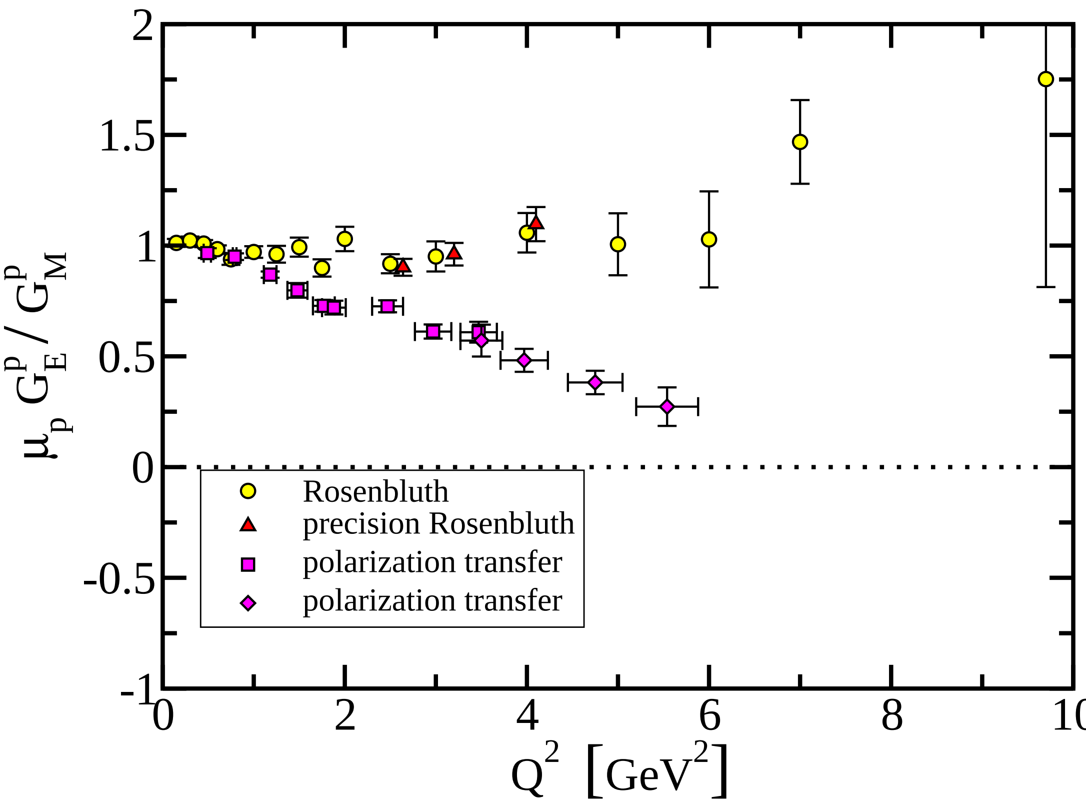


Form Factor Ratio: GE/GM



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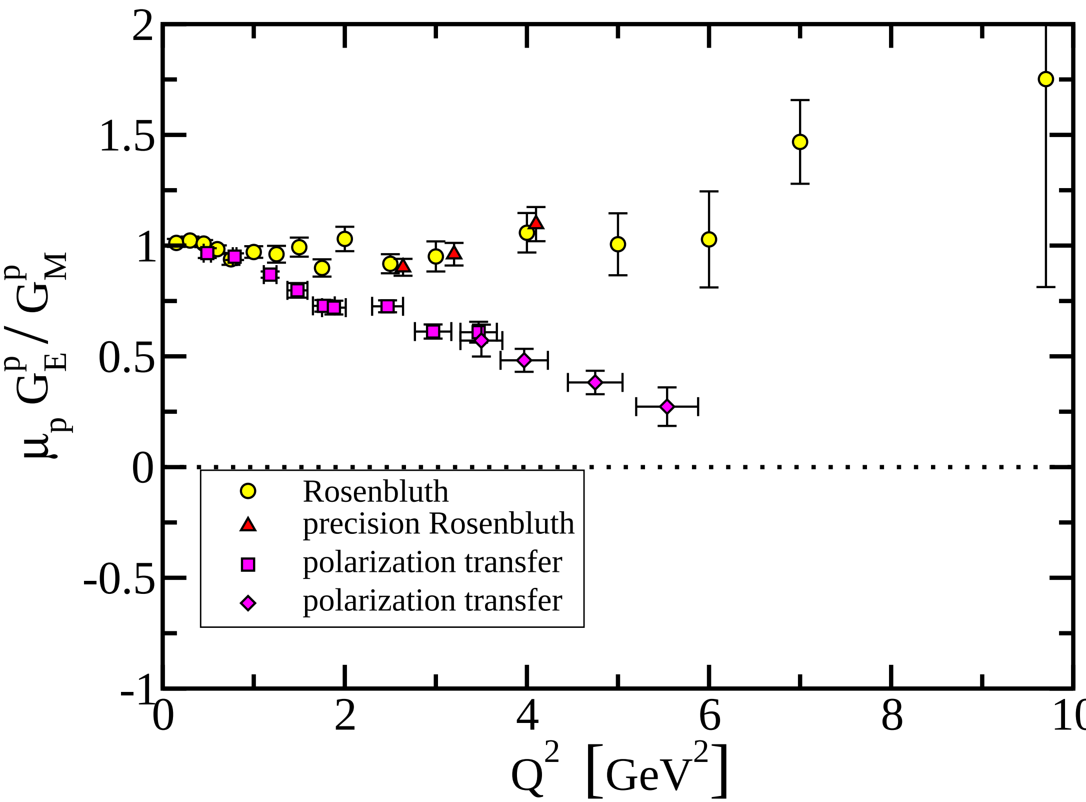
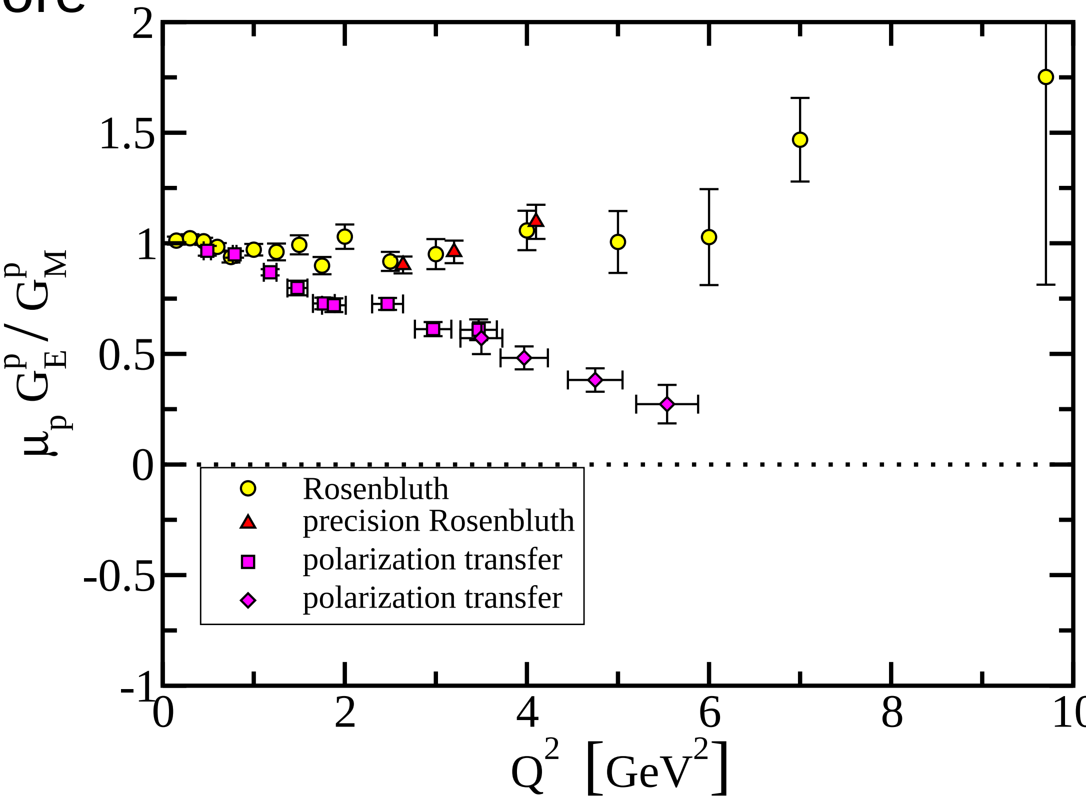


- Combine these elements . . .
  - Dressed-Quark Core



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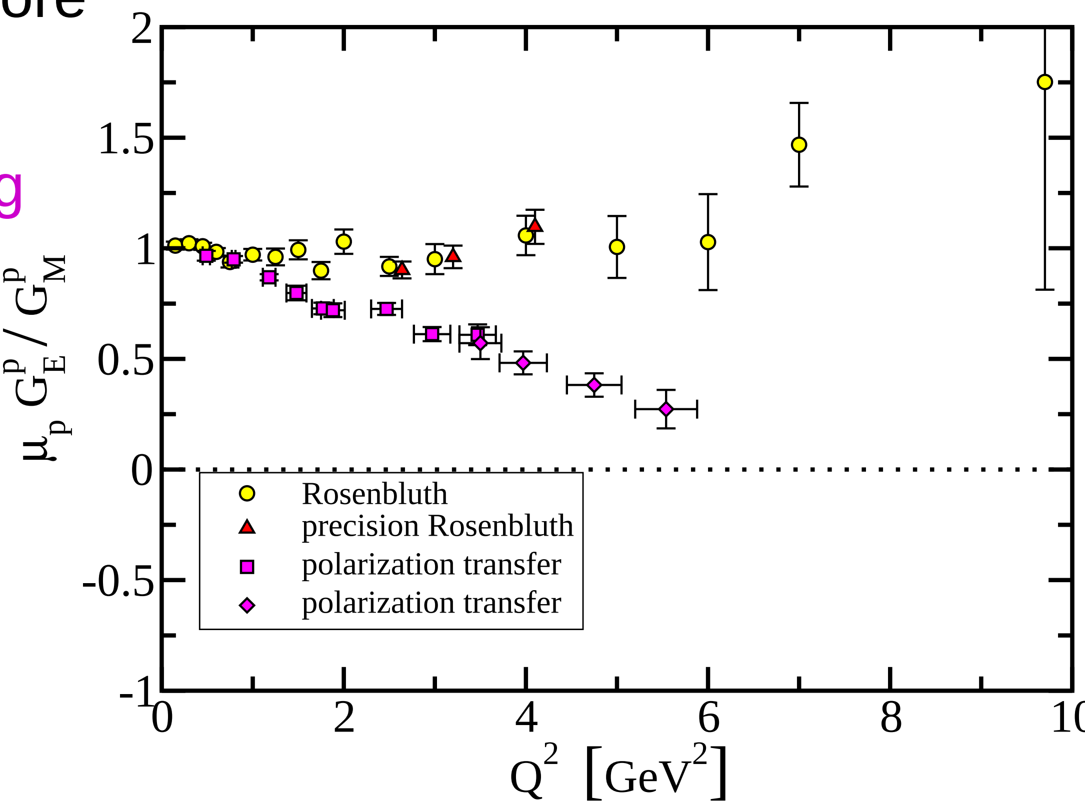

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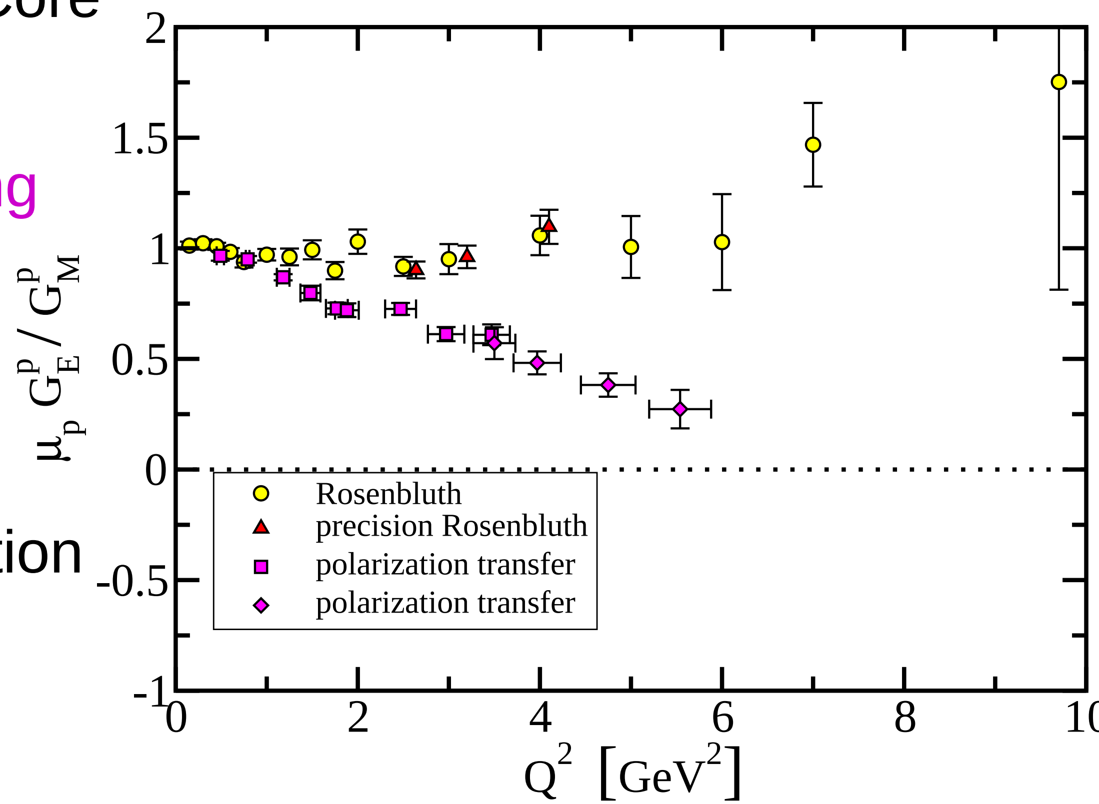

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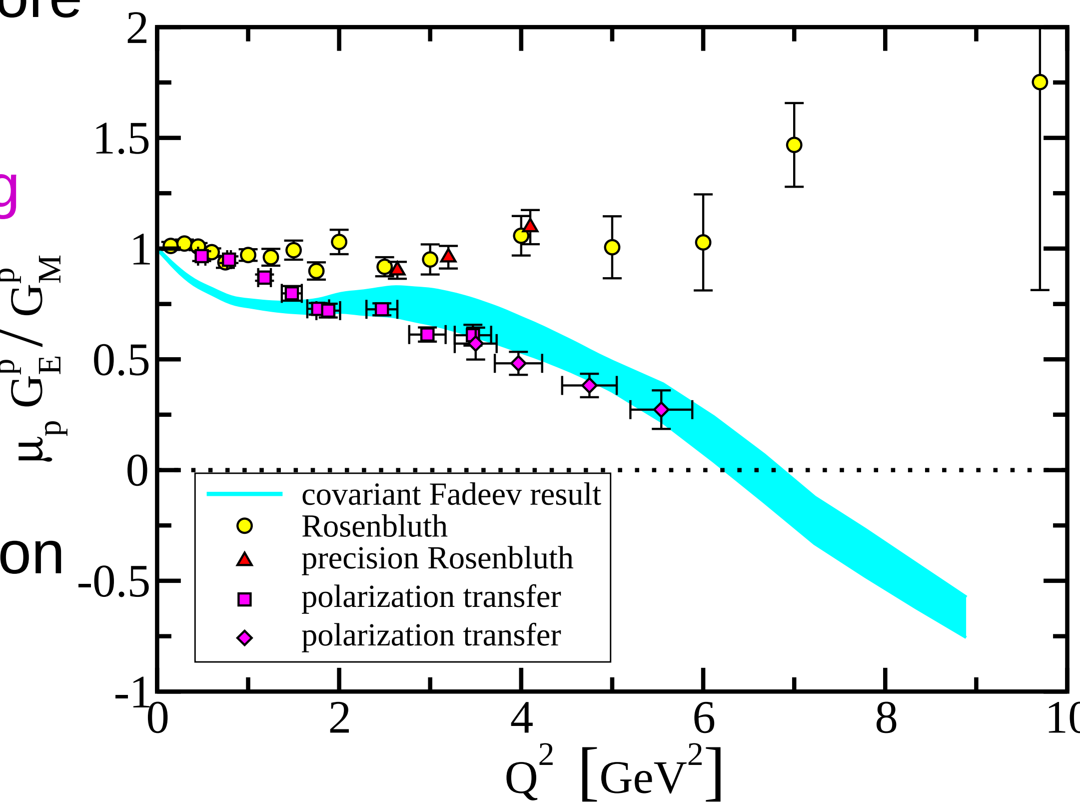

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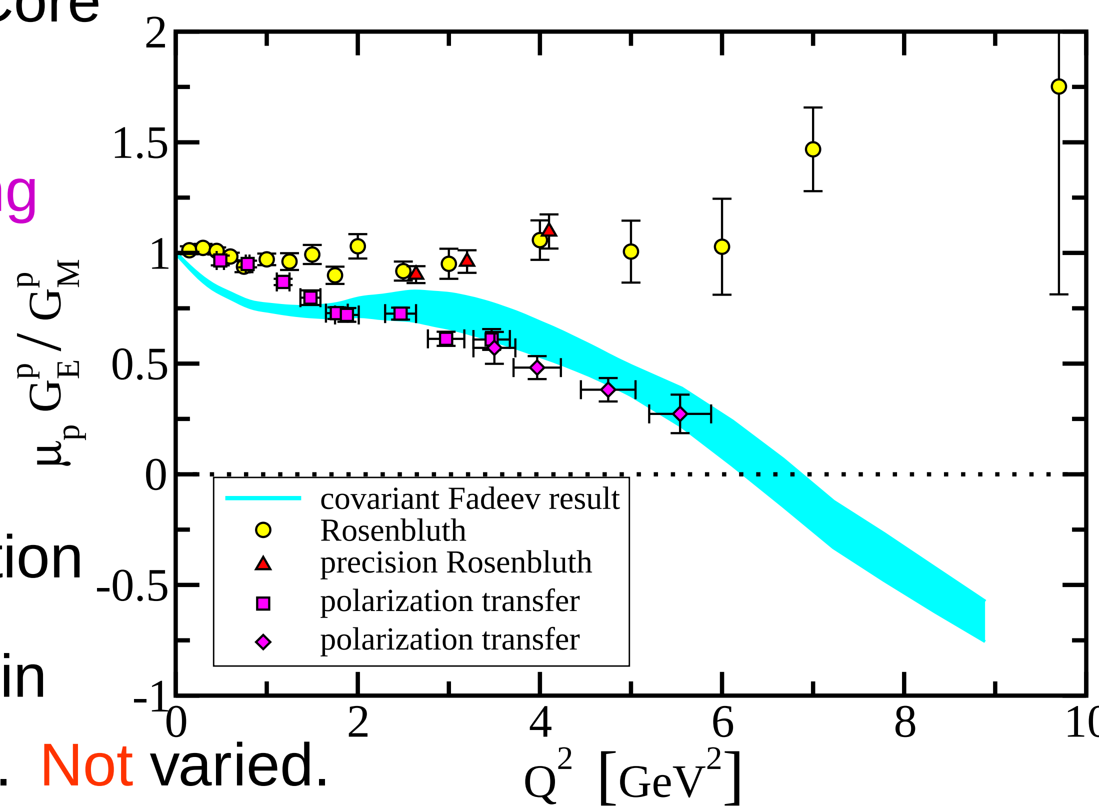

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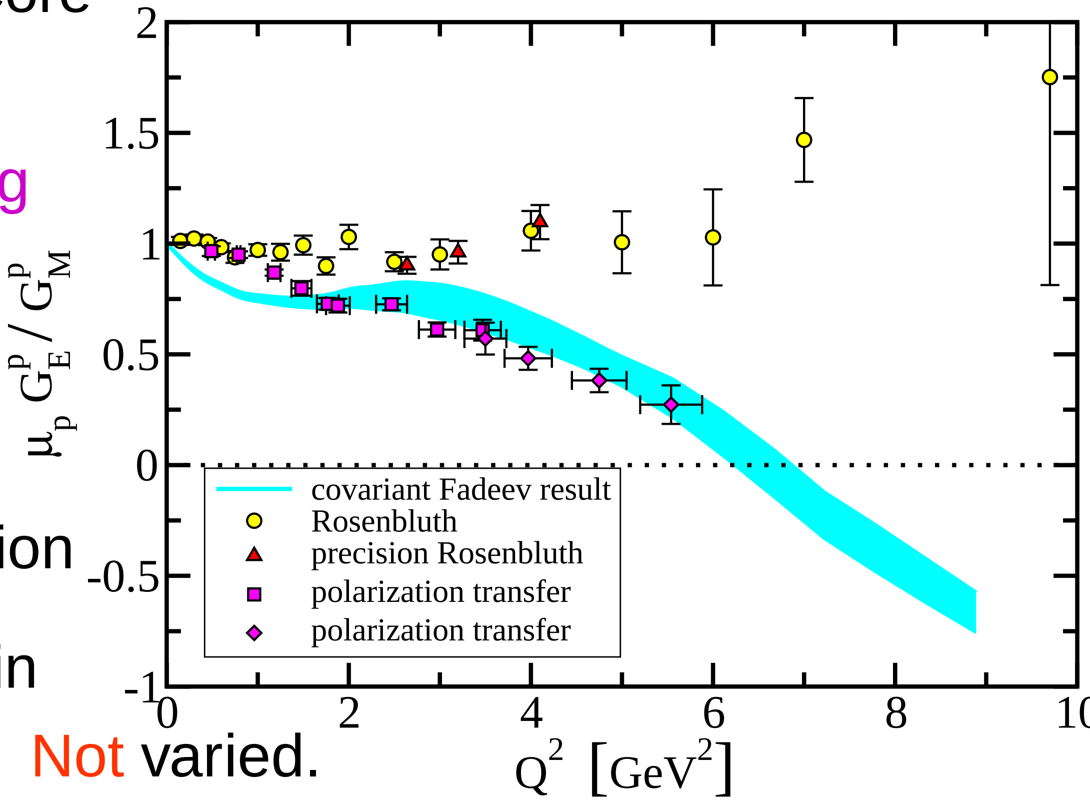

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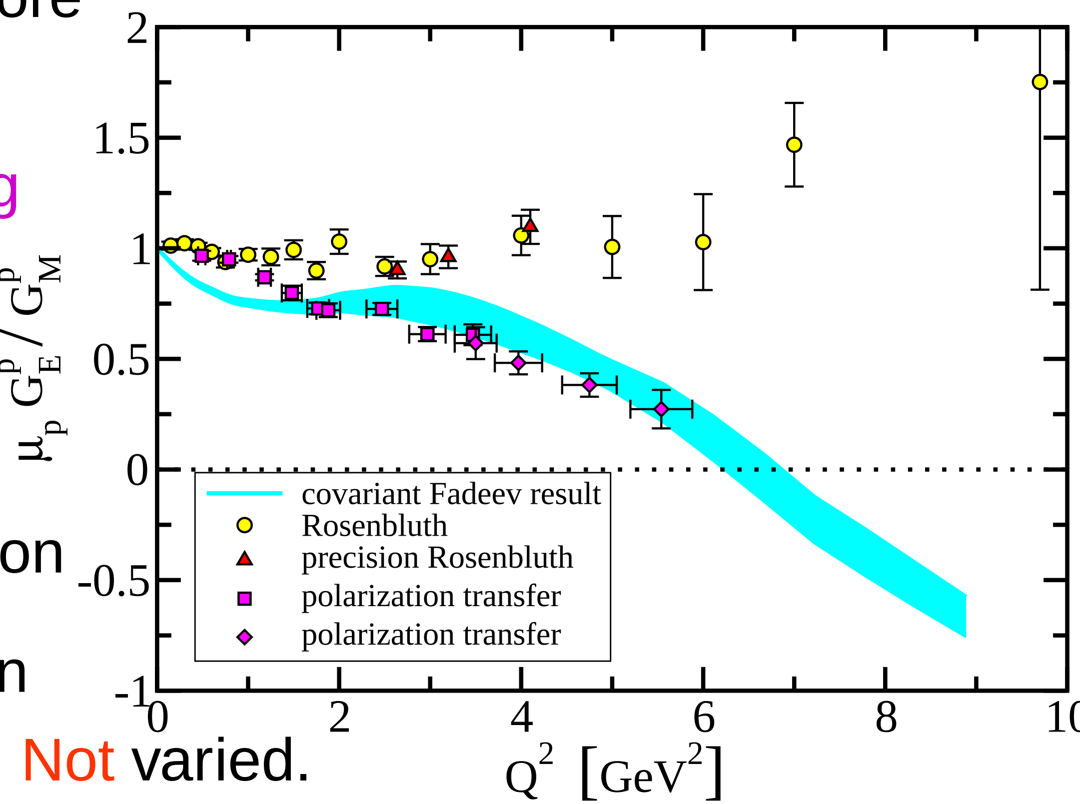

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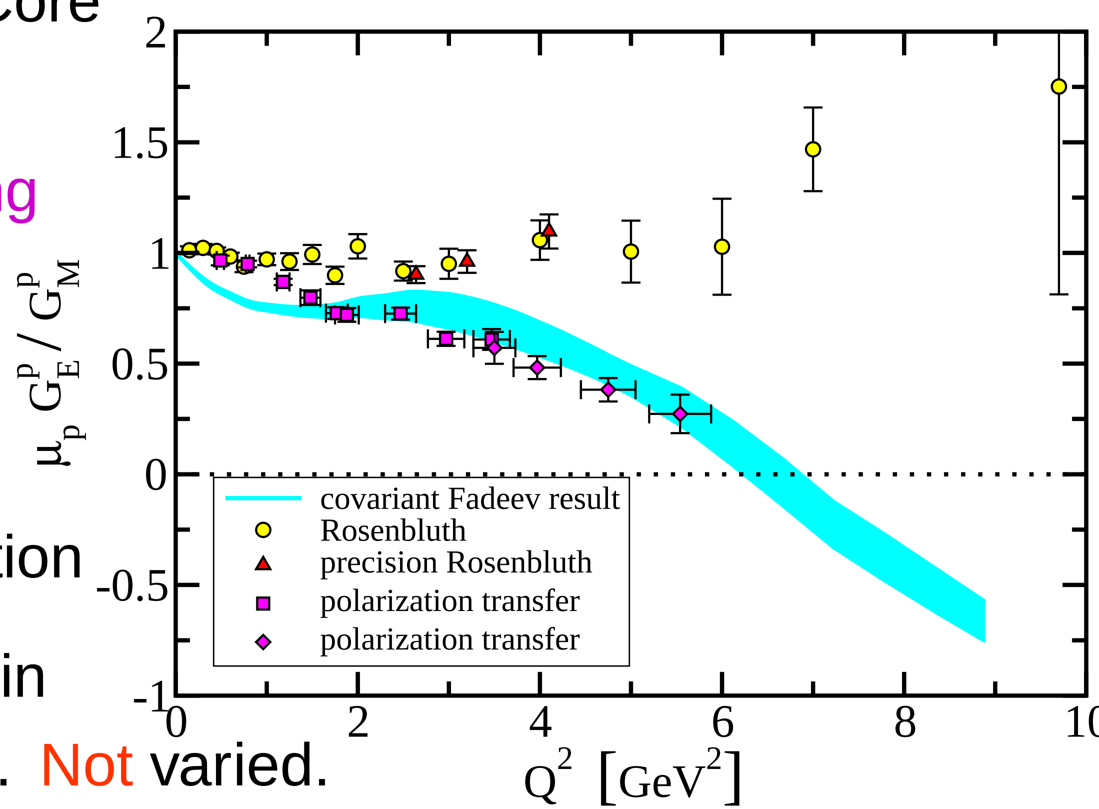

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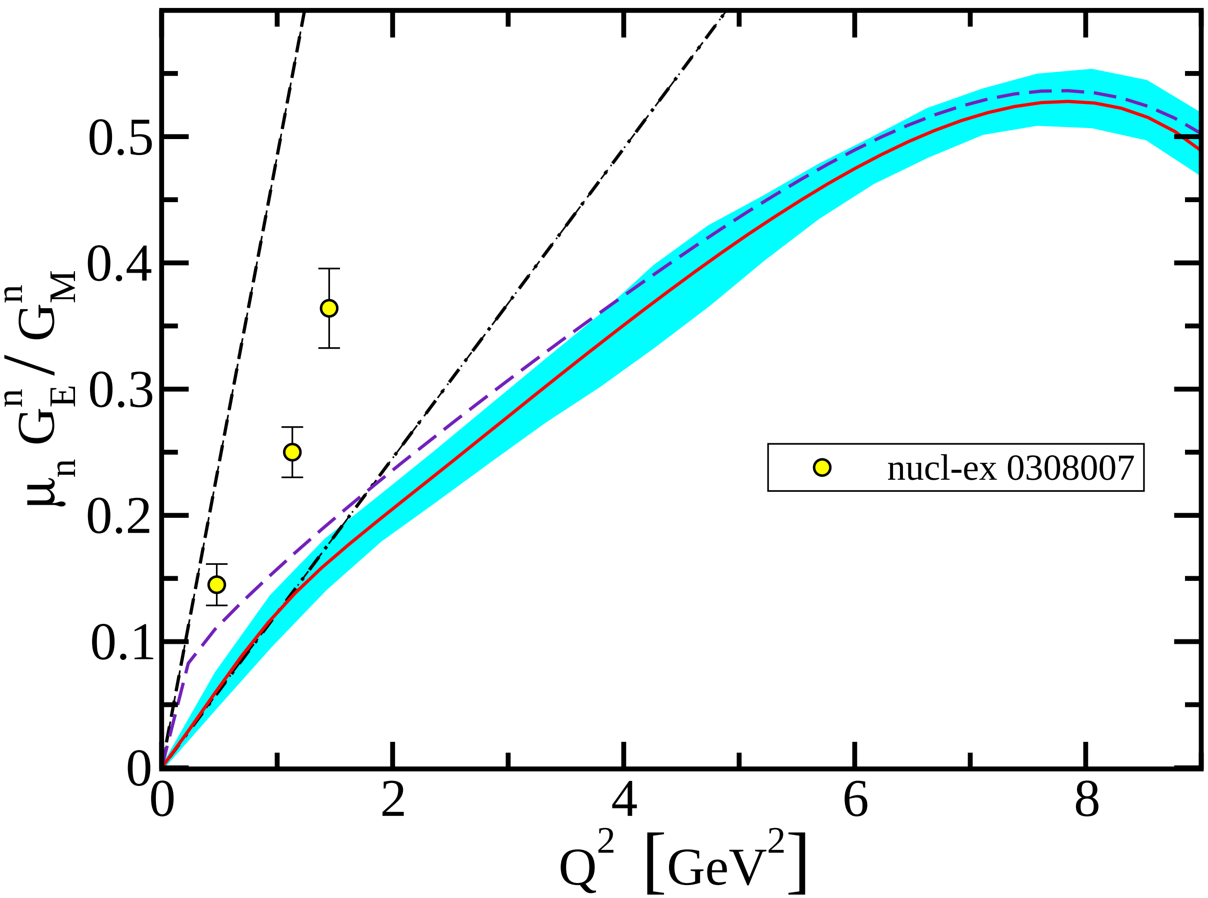
  - Predict Zero at $Q^2 \approx 6.5$GeV^2

Neutron Form Factors



Neutron Form Factors

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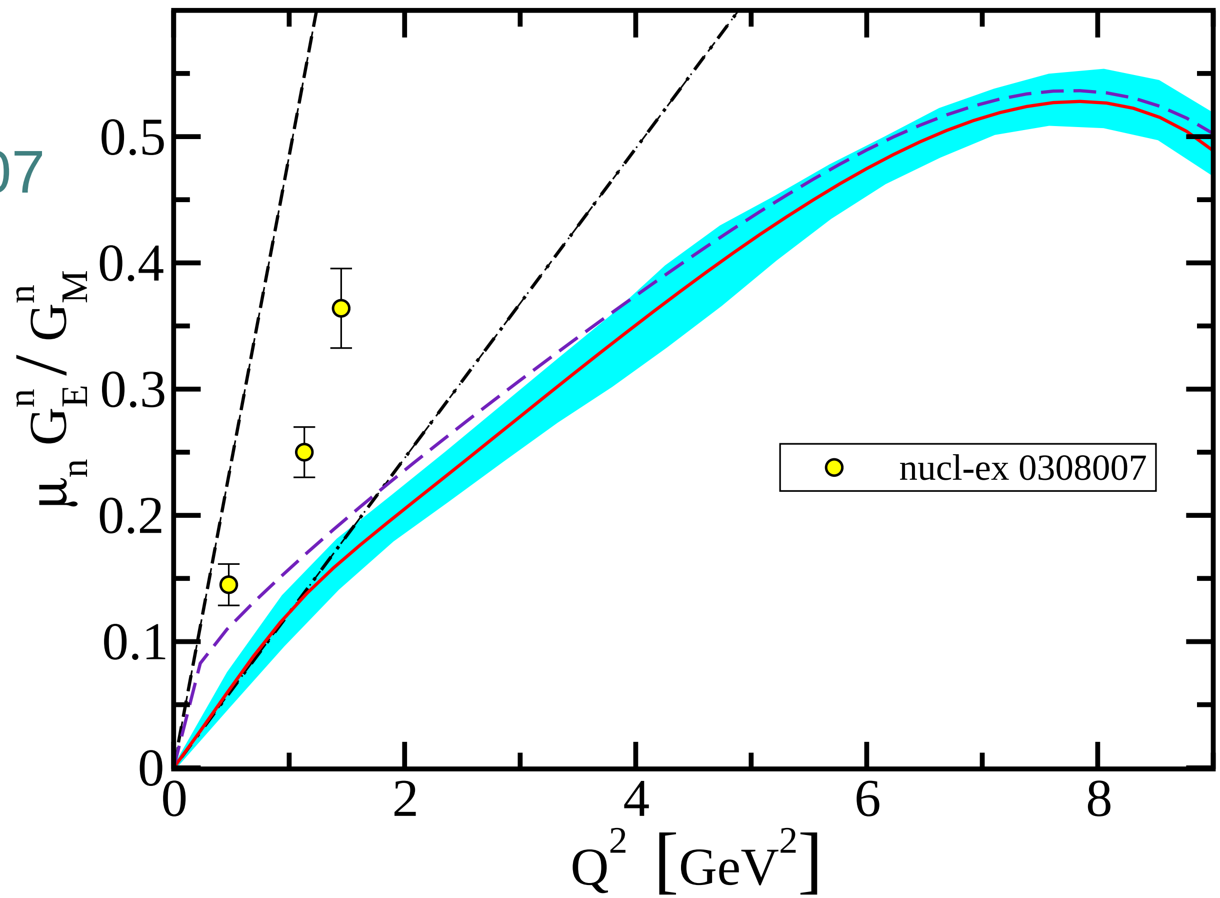
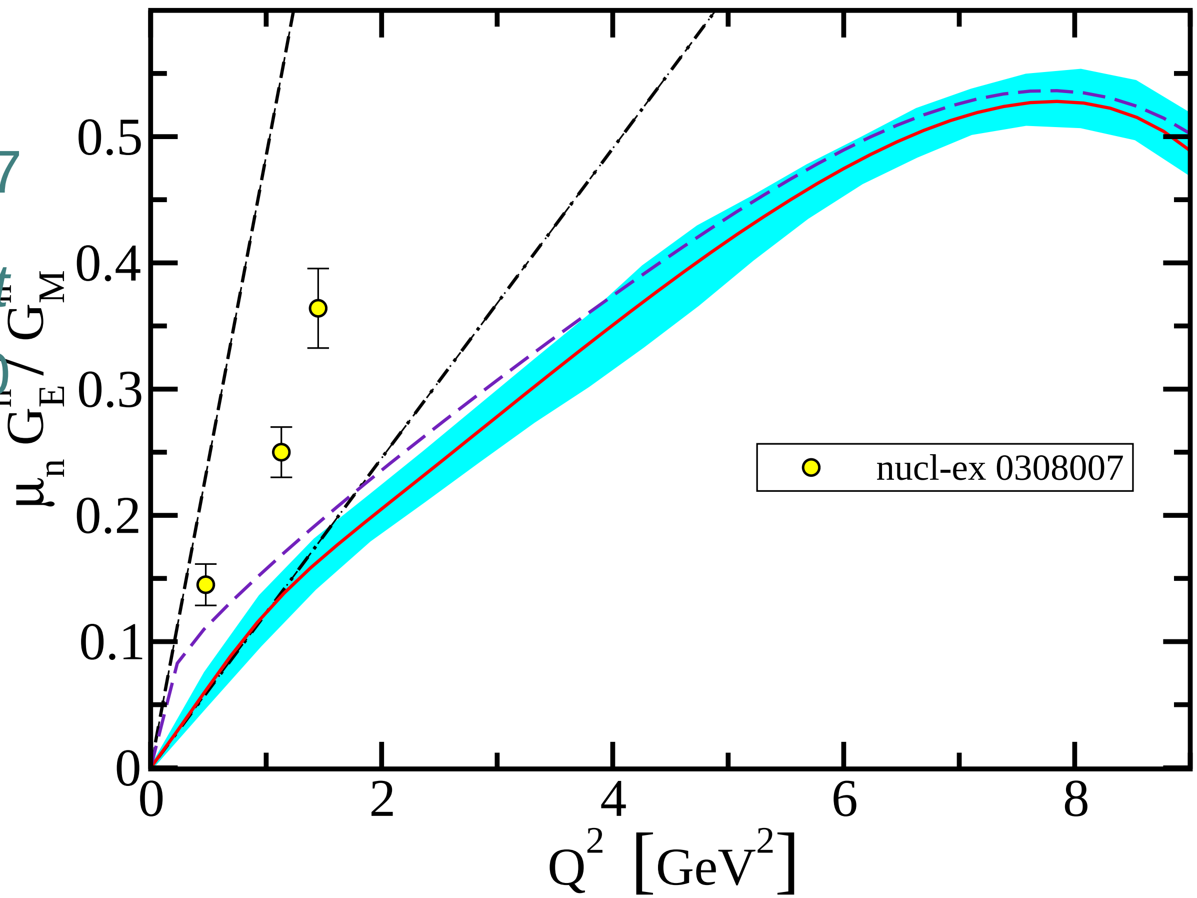


- Expt. Madey, *et al.* nu-ex/0308007
- Calc. Bhagwat, *et al.* nu-th/0610080
- $\mu_p \frac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\frac{r_n^2}{6} Q^2$

Valid for $r_n^2 Q^2 \lesssim 1$



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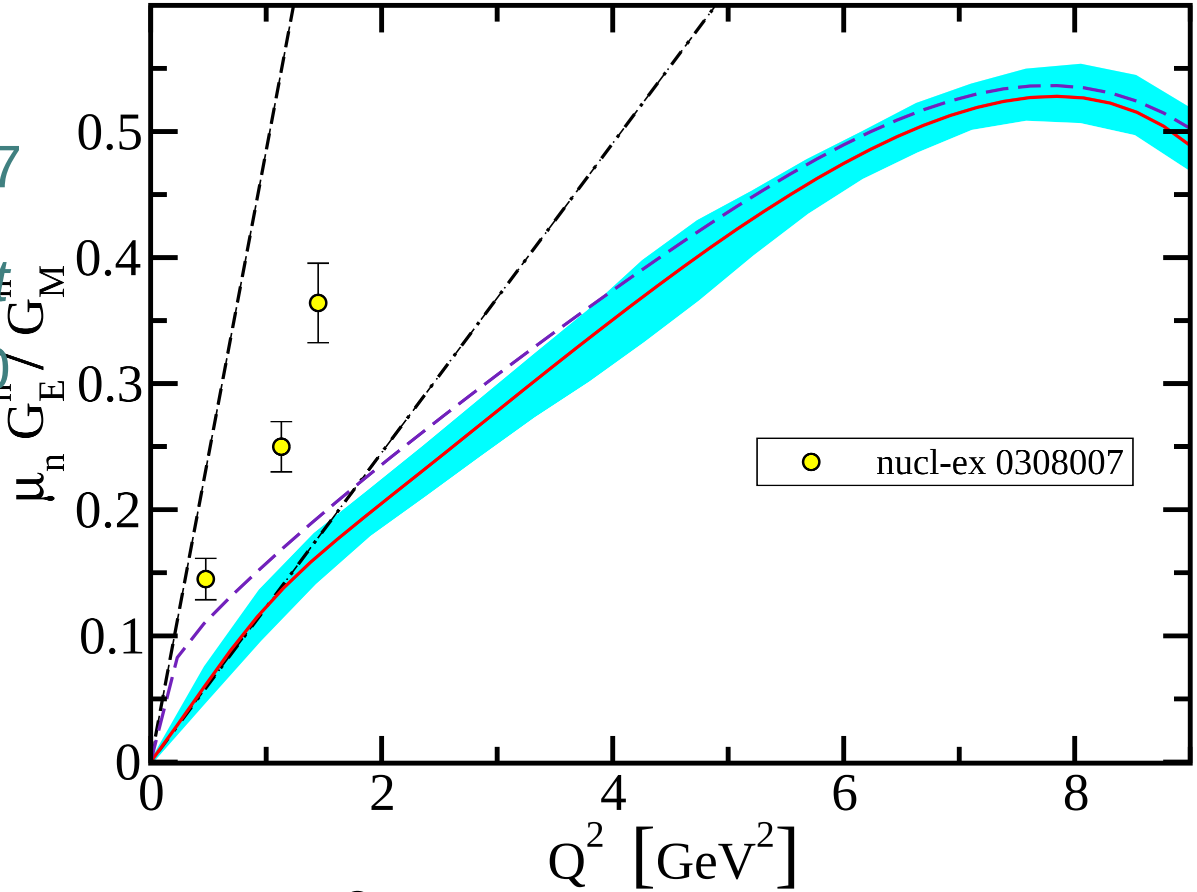

- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4\,\mathrm{GeV}^2$ is being analysed (JLab E02-013)

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- DSEs . . . contemporary tool that describes and explains these phenomena, and connects them with prediction of observables

Office of Science
U.S. DEPARTMENT OF ENERGY
Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars
Argonne NATIONAL LABORATORY

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Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

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Detmold *et al*., nu-th/0202082

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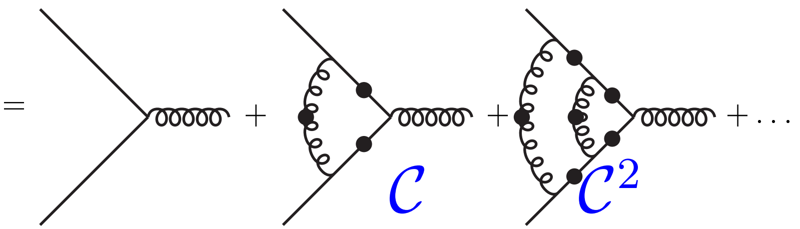
- Coupling-modified dressed-ladder vertex

$\Gamma^a_\mu(k,p) \;=$



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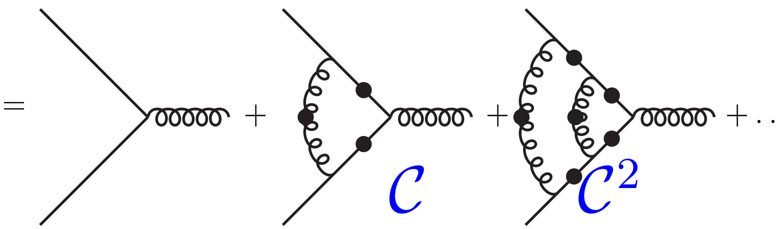
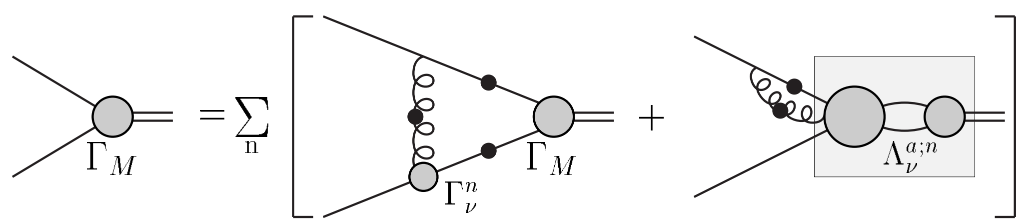
$\Gamma^a_\mu(k,p) =$ 

- BSE consistent with vertex



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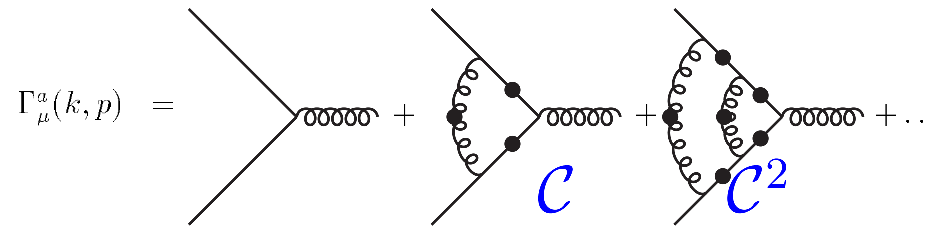

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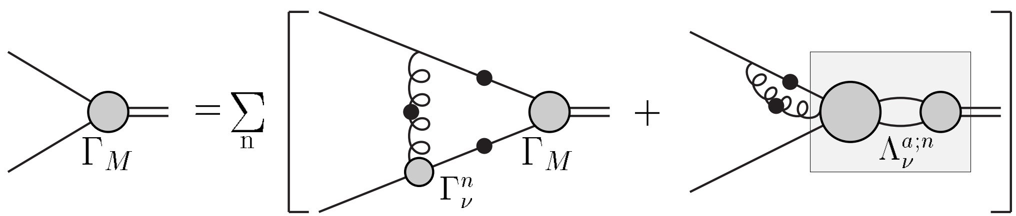
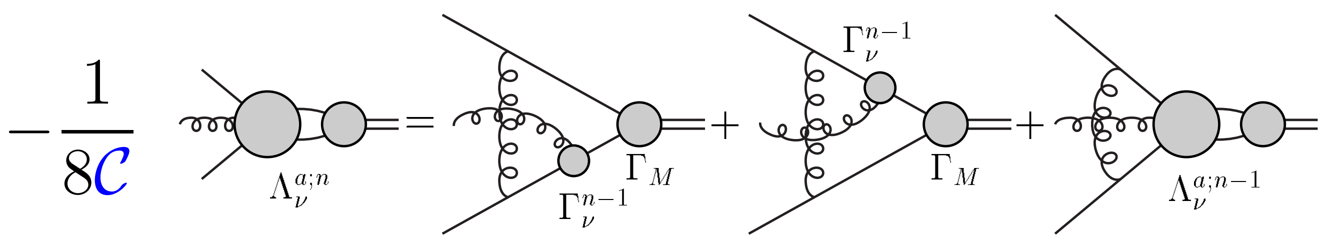


- Bethe-Salpeter kernel . . . recursion relation

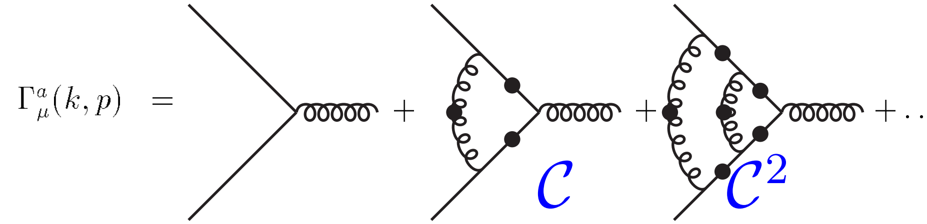


Colour-singlet Bethe-Salpeter equation

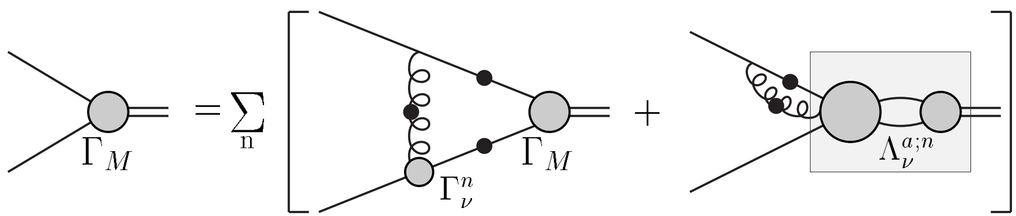
Detmold *et al.*, nu-th/0202082

Bhagwat, *et al.*, nu-th/0403012

- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex



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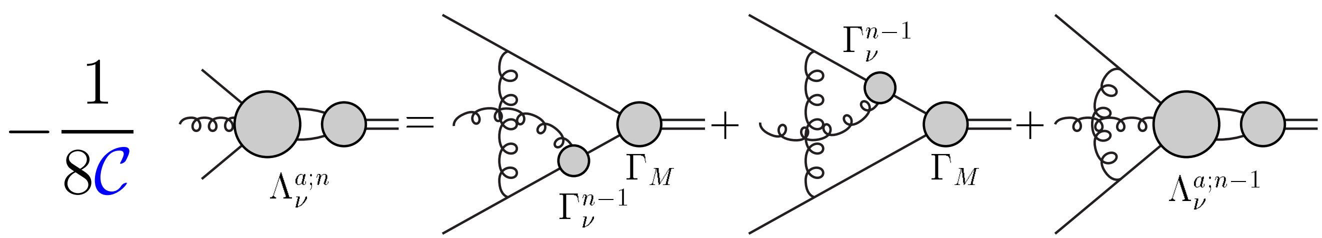

- Kernel necessarily non-planar,

  even with planar vertex

π and ρ mesons

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π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

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- ALL π-ρ mass splitting present in chiral limit

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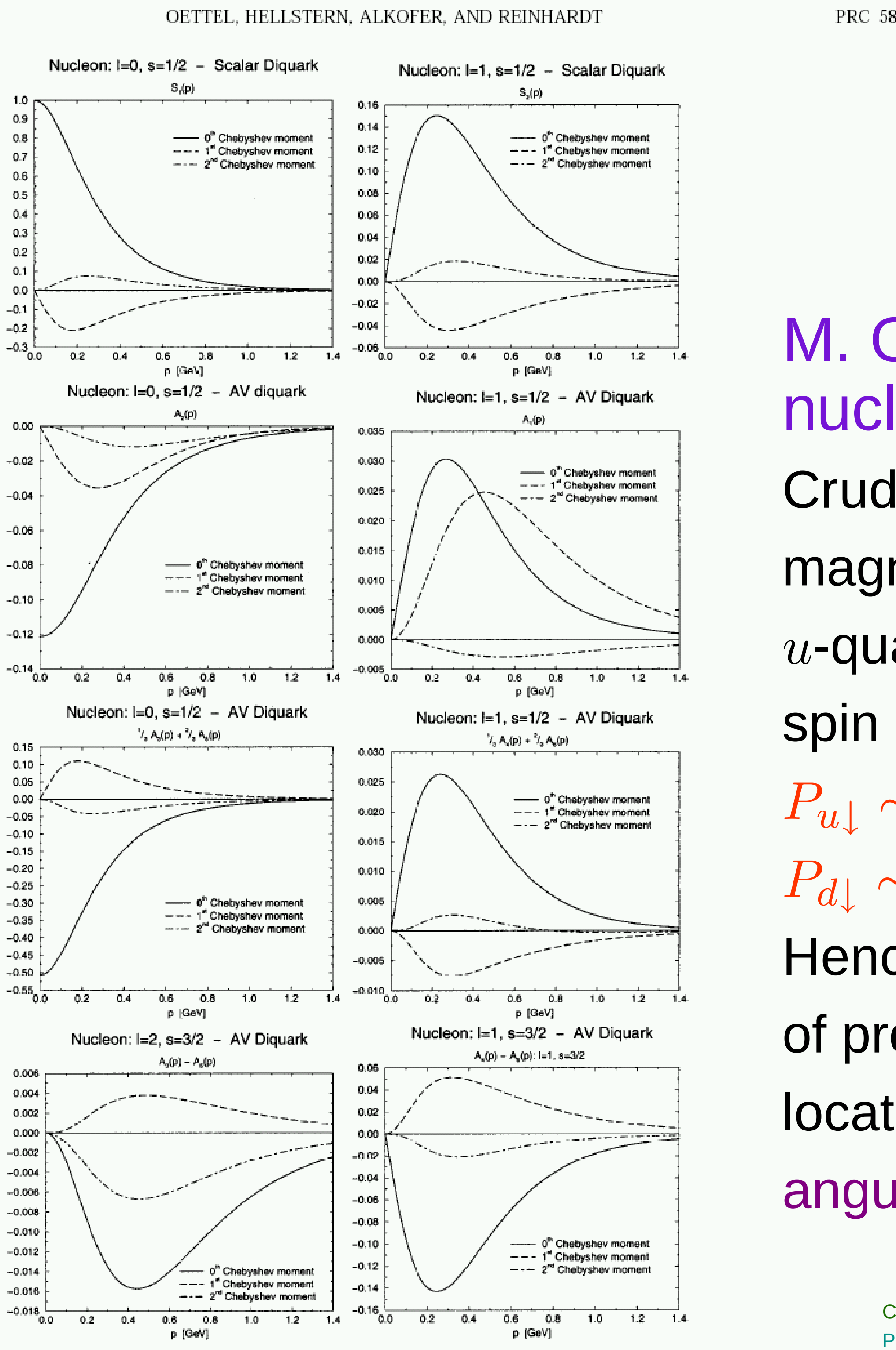
  – two loop, accurate to 4%



Angular Momentum Rest Frame

M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Deep-inelastic scattering

Deep-inelastic scattering

- Looking for Quarks

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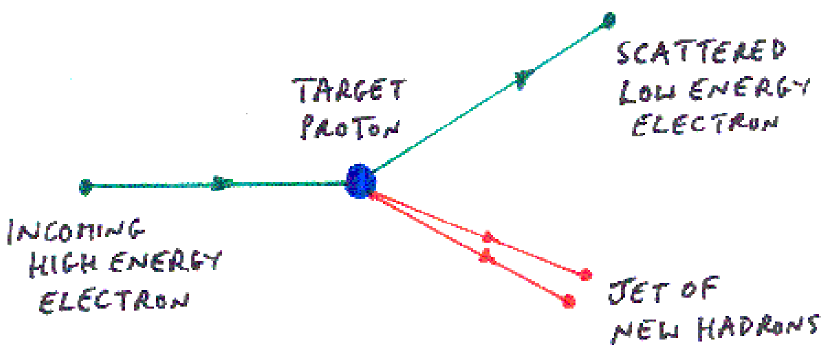
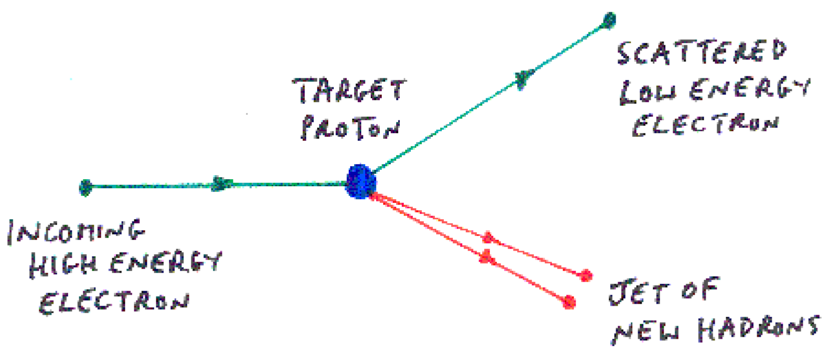
Deep-inelastic scattering



- Looking for Quarks

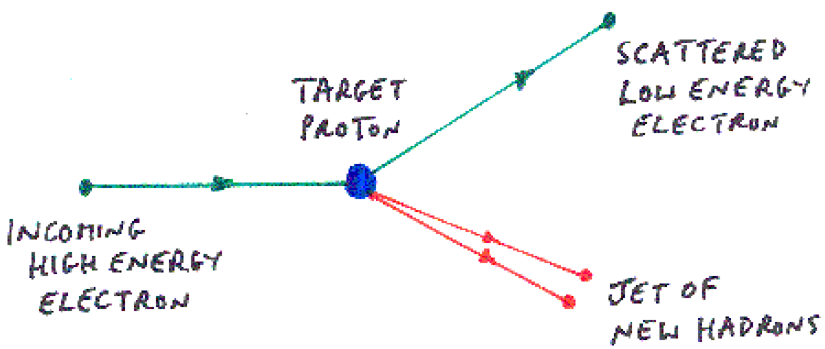
Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:
  Discovery of Quarks at SLAC

Deep-inelastic scattering



- Looking for Quarks

- $\boxed{\text{Signature Experiment}}$ for QCD:
  Discovery of Quarks at SLAC

- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

Pion's valence quark distn

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Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!

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Pion's valence quark distn

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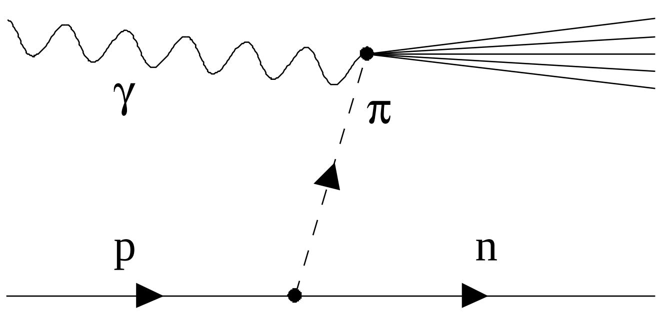
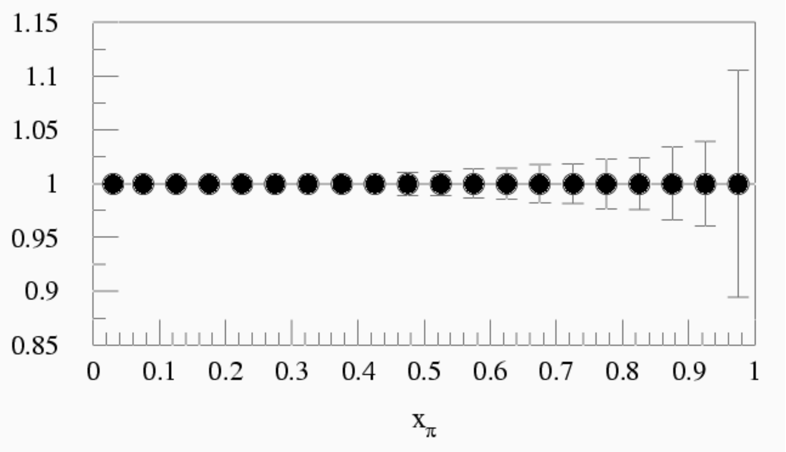
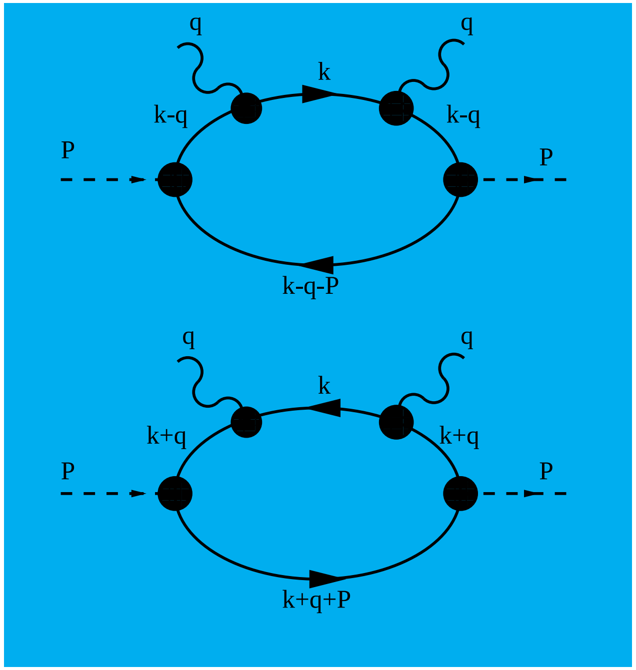
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+ \mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

$e^-_{5\,\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"





Handbag diagrams



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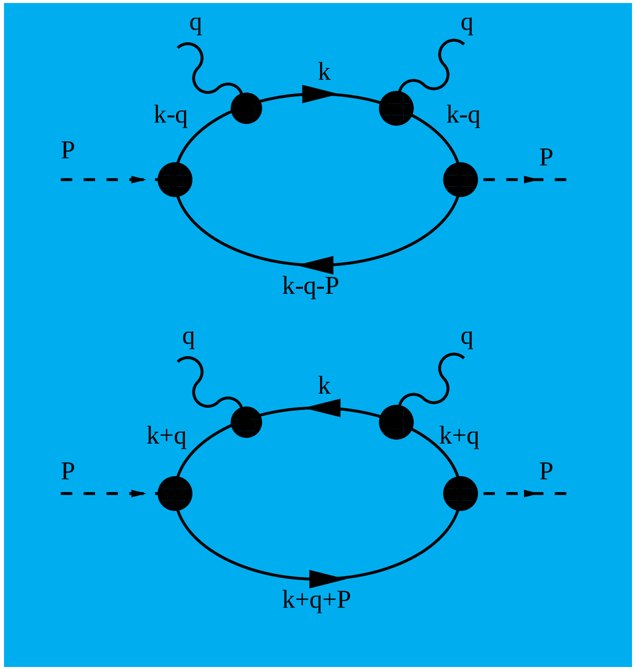

$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

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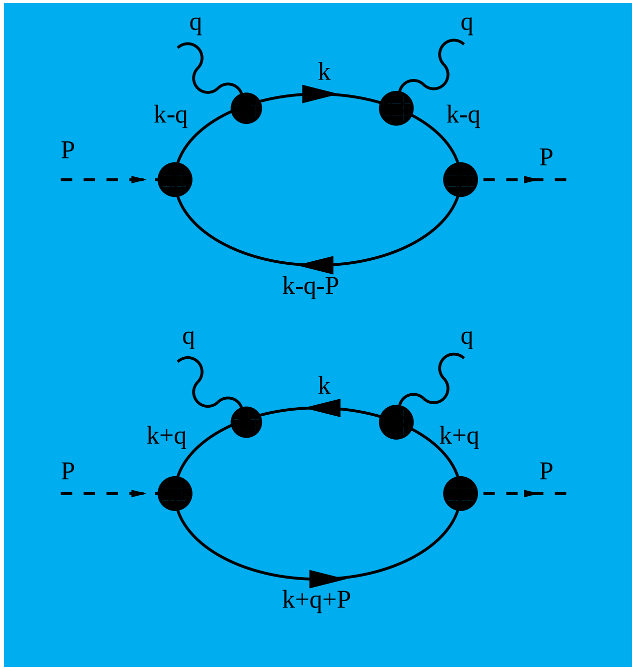
Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.

Numerous algebraic simplifications



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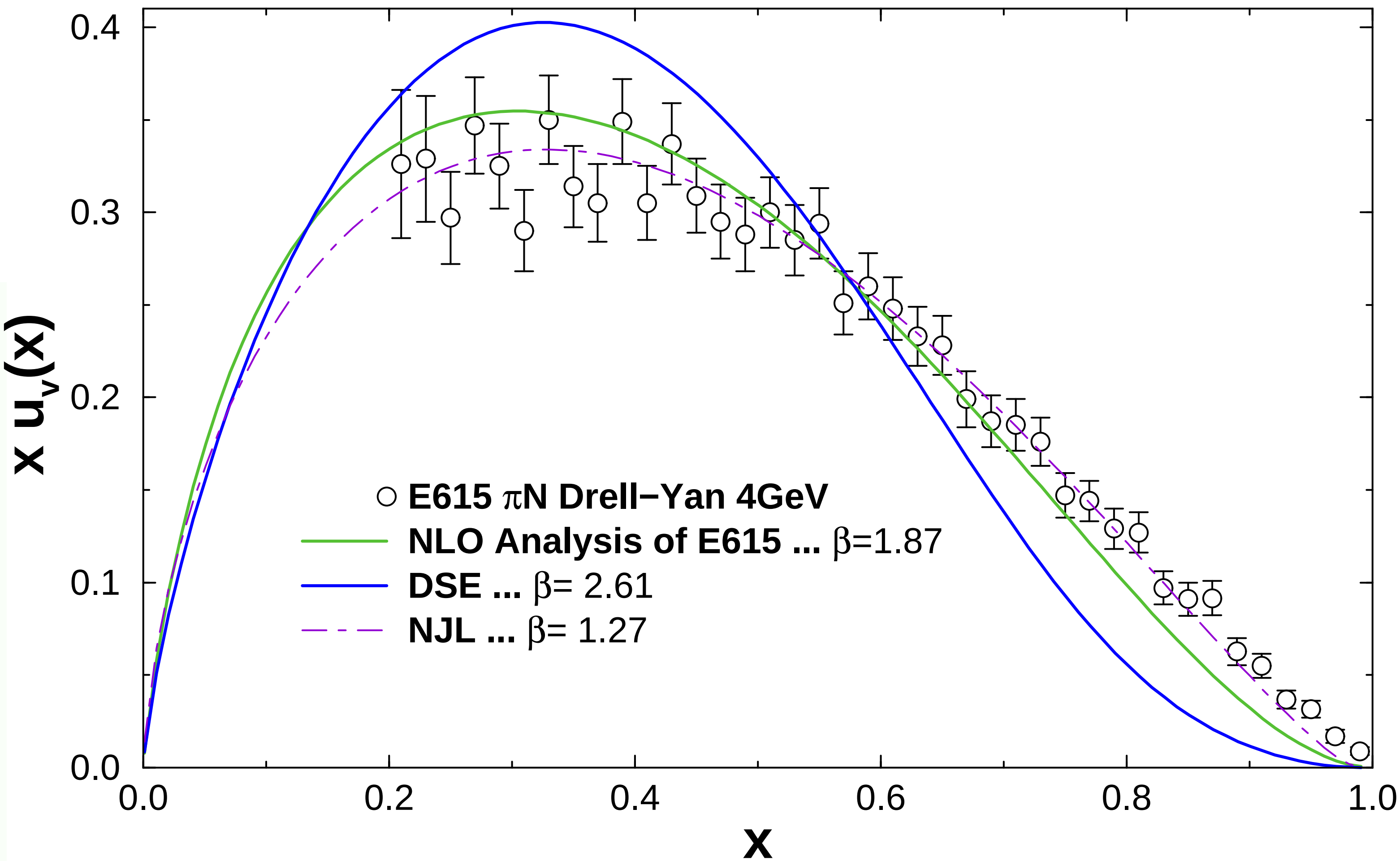
Extant theory vs. experiment

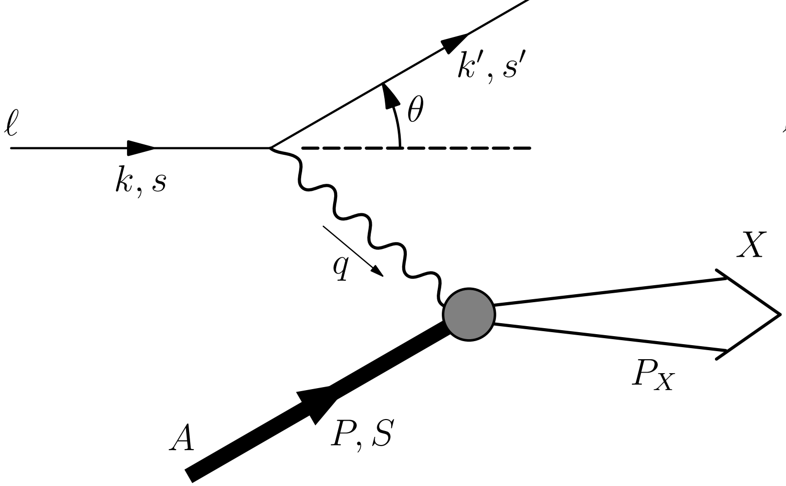
K. Wijersooriya, P. Reimer and R. Holt,
nu-ex/0509012 ... Phys. Rev. C (Rapid)



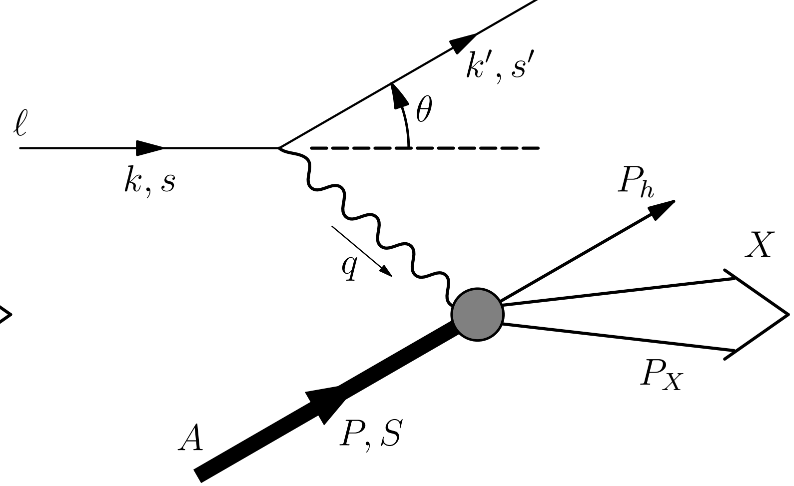
Nucleon's Quark Distribution Functions

Nucleon's Quark Distribution Functions

DIS



SI–DIS



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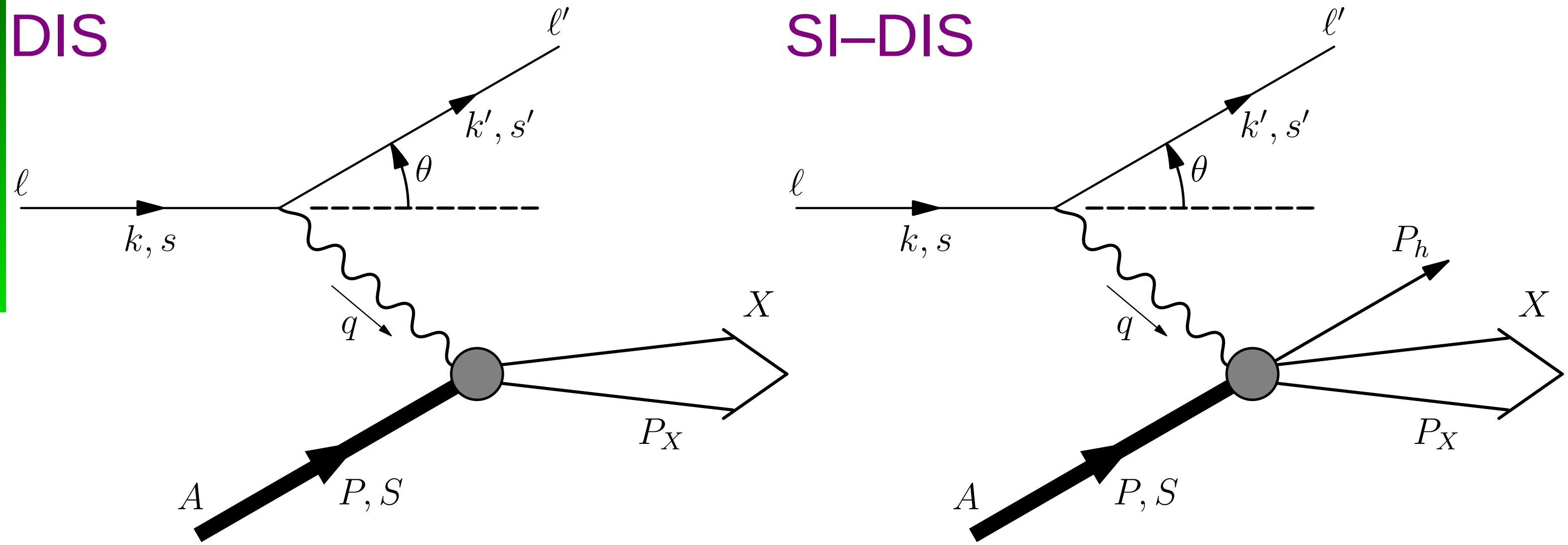

- Three twist-2 parton distributions $(k_\perp = 0)$:
  - Spin-Independent: $q(x)$
  - Helicity: $\Delta q(x)$
  - Transversity: $\Delta_T q(x)$
- All distributions have probability interpretation.
- By definition, contain essentially non-perturbative information about a given process.

Definition and Sum Rules

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Definition and Sum Rules

- Light-cone Fourier transforms :

$$\Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p,s|\overline{\psi}_q(0)\gamma^+\gamma^1\gamma_5\psi_q(\xi^-)|p,s\rangle_c$$

$$q(x) = \langle\gamma^+\rangle, \qquad \Delta q(x) = \langle\gamma^+\gamma_5\rangle$$

- Related to the nucleon axial & tensor charges via

$$g_A = \int dx[\Delta u(x) - \Delta d(x)], \quad g_T = \int dx[\Delta_T u(x) - \Delta_T d(x)],$$

- Must satisfy: positivity constraints and Soffer bound

$$\Delta q(x), \Delta_T q(x) \leqslant q(x), \quad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

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Ian Cloët

JLab, now ANL

Ian Cloët
JLab, now ANL

Once more on the one that got away.

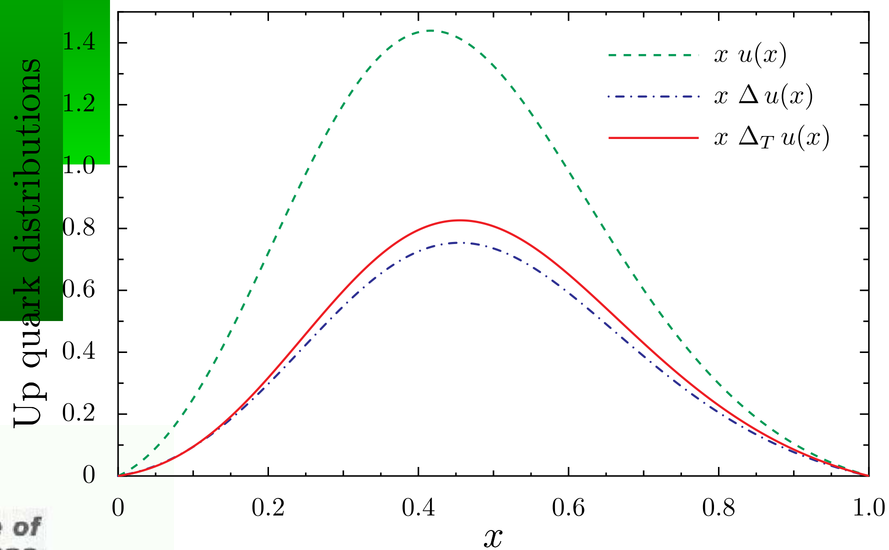
Model predictions

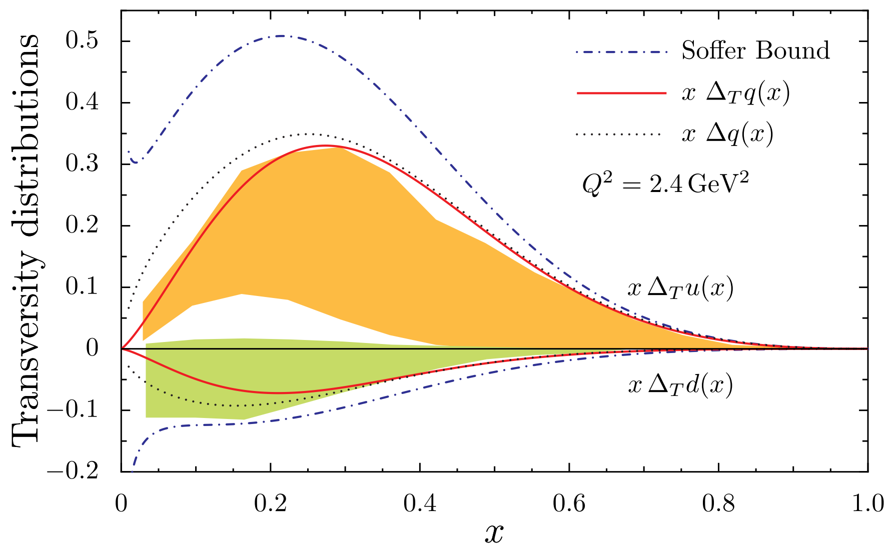
Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

Model predictions

- Simplified Faddeev equation





- Satisfy: Soffer bound, baryon & momentum SRs.

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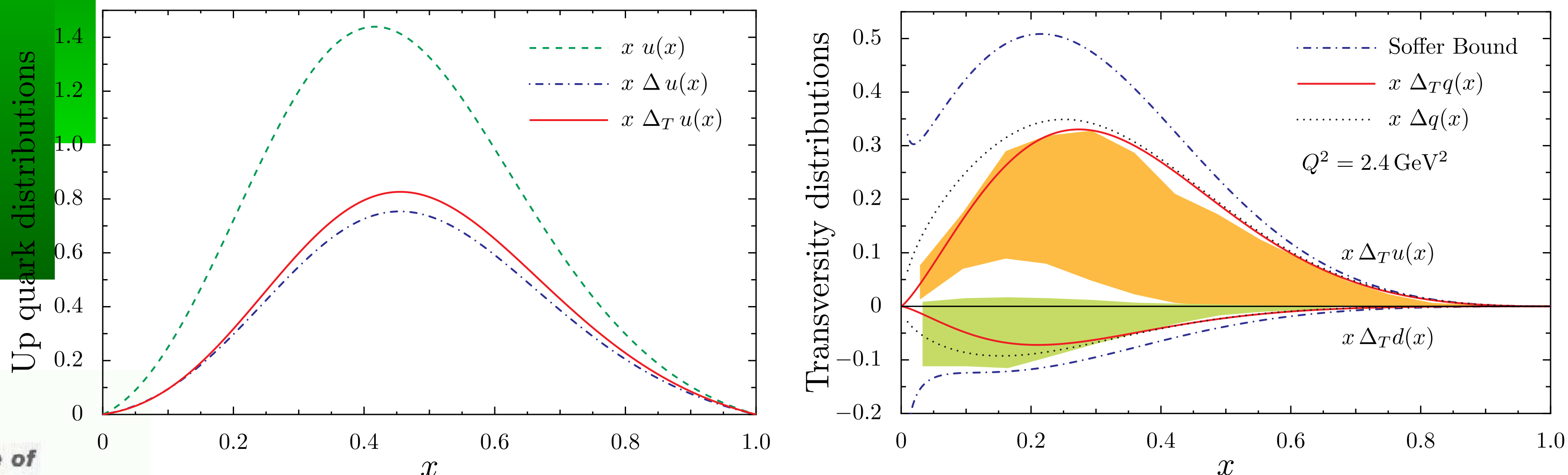

- Satisfy: Soffer bound, baryon & momentum SRs.
- Moments at $Q^2 = 0.16\,\mathrm{GeV}^2$:

$$\Delta u = 0.97,\quad \Delta d = -0.30 \quad \Longrightarrow \quad g_A = 1.267$$
$$\Delta_T u = 1.04,\, \Delta_T d = -0.24 \quad \Longrightarrow \quad g_T = 1.28$$

Model constraint

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Cloët, Bentz, Thomas
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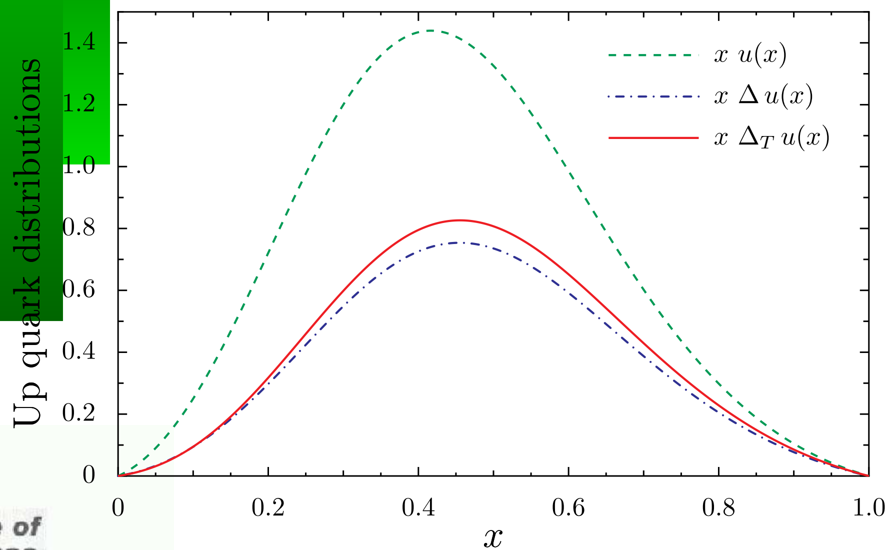

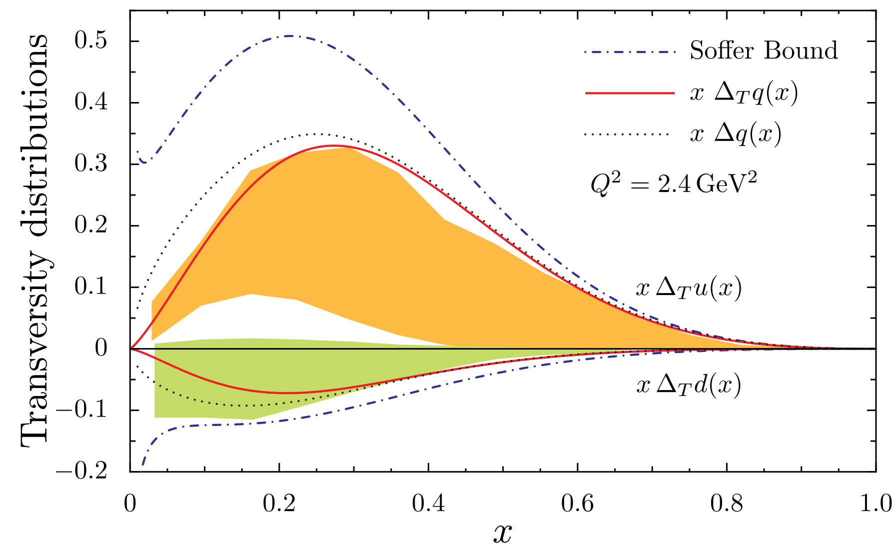

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- $\Delta q(x) \sim \Delta_T q(x)$ in valence region for $Q^2 \lesssim 10\,$GeV^2